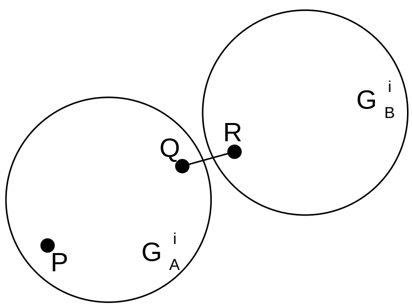
Appendix A
Proof of Lemma 1

Lemma 1: A node from a level-i group can reach a node in any adjacent level-i group in at most 3^i hops.

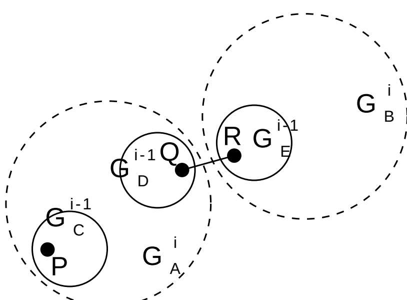
Proof: The proof is performed by induction.

Basis: $i = 0$. Let's take two arbitrary adjacent level-0 groups: G_A^0 and G_B^0. From Property 1, $G_A^0 = \{A\}$ and $G_B^0 = \{B\}$. G_A^0 and G_B^0 are adjacent, thus A and B are neighbors, that is, A can reach B in $1 = 3^0$ hop. Since we chose G_A^0 and G_B^0 arbitrarily, the lemma is true for $i = 0$.

Inductive step: $i = k+1$ (where $k \geq 0$). We assume that the lemma holds for all levels $\leq k$. Let's take two arbitrary adjacent level-i groups G_A^i and G_B^i, and an arbitrary node $P \in G_A^i$. Let R denote a node in G_B^i that has a neighbor Q such that $Q \in G_A^i$ (existence of Q is guaranteed by the definition of adjacent groups).

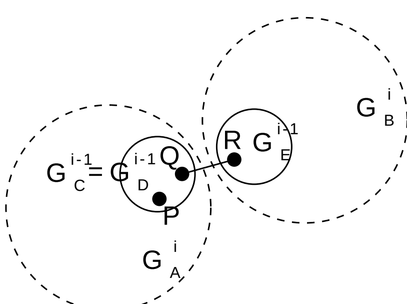


Consider level-i subgroups, that is, G_C^{i-1}, G_D^{i-1}, and G_E^{i-1}, such that $P \in G_C^{i-1}$, $Q \in G_D^{i-1}$, and $R \in G_E^{i-1}$.



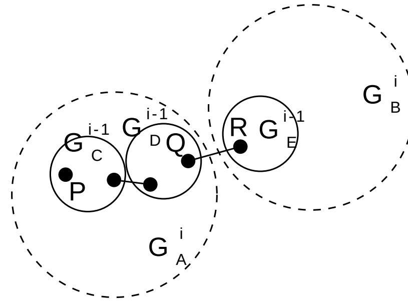
We have the following three situations:

1): $C = D$ (group G_C^{i-1} is adjacent to group G_E^{i-1}).



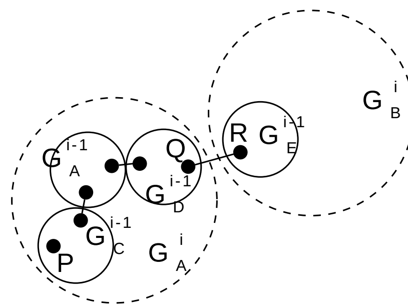
In this case, from the inductive assumption P can reach some node from G_E^{i-1} (hence in G_B^i) in at most $3^{i-1} < 3^i$ hops.

2): G_C^{i-1} is adjacent to G_D^{i-1}.



In this case, from the inductive assumption P can get to some node in G_D^{i-1} in a most 3^{i-1} hops and any node from G_D^{i-1} can get to a node from G_E^{i-1} in at most 3^{i-1} hops. Consequently, P can get to some node from G_E^{i-1} (hence in G_B^i) in at most $2 \cdot 3^{i-1} < 3^i$ hops.

3): G_C^{i-1} is not adjacent to G_D^{i-1} (but from Property 4, G_C^{i-1} and G_D^{i-1} are both adjacent to G_A^{i-1}).



In this case, from the inductive assumption P can get to some node in G_A^{i-1} in at most 3^{i-1} hops. Likewise, any node from G_A^{i-1} can get to a node from G_D^{i-1} in at most 3^{i-1} hops and any node from G_D^{i-1} can get to a node from G_E^{i-1} in at most 3^{i-1} hops. Therefore, P can get to some node from G_E^{i-1} (hence in G_B^i)in at most $3 \cdot 3^{i-1} = 3^i$ hops.

Consequently P can get to a node from G_B^i in at most 3^i hops. Since we chose P arbitrarily, any node from G_A^i can get to some node from G_B^i in at most 3^i. Because G_A^i and G_B^i were also chosen arbitrarily, the lemma is true for $i = k+1$.

By applying mathematical induction to the basis and the inductive step, we proved the lemma for all i. Moreover, 3^i is a tight bound, that is, it is reachable for some configurations. □

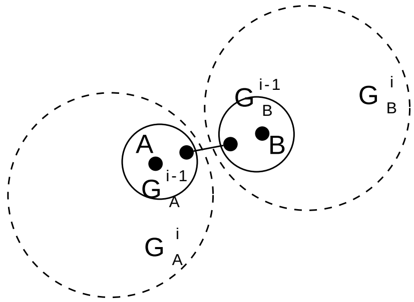
Appendix B
Proof of Lemma 2

Lemma 2: The distance between the head nodes of two adjacent level-i groups is at most 3^i hops.

Proof: The proof is performed by induction. Let $d(A,B)$ denote the distance in hops between nodes A and B.

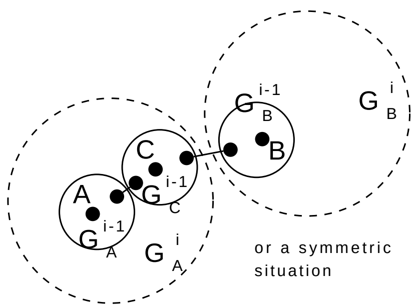
Basis: $i = 0$. Let's take two arbitrary adjacent level-0 groups: G_A^0 and G_B^0. From Property 1, $G_A^0 = \{A\}$ and $G_B^0 = \{B\}$. G_A^0 and G_B^0 are adjacent, thus A and B are neighbors, that is, $d(A,B) = 1 = 3^0$. As we chose G_A^0 and G_B^0 arbitrarily, the lemma is true for $i = 0$.

Inductive step: $i = k+1$ (where $k \geq 0$). We assume that the lemma holds for all levels $\leq k$. Let's take two arbitrary adjacent level-i groups G_A^i and G_B^i. We have three possible situations: *1): G_A^{i-1} is adjacent to G_B^{i-1}.*



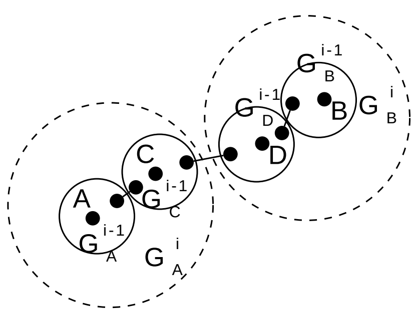
In this case, from the inductive assumption, $d(A,B) \leq 3^{i-1} < 3^i$.

2): There exists G_C^{i-1} such that it belongs to G_A^i or G_B^i and it is adjacent to both G_A^{i-1} and G_B^{i-1}.



In this case, $d(A,B) \leq d(A,C) + d(B,C)$. From the inductive assumption $d(A,C), d(B,C) \leq 3^{i-1}$, thus $d(A,B) \leq 2 \cdot 3^{i-1} < 3^i$.

3): There exist G_C^{i-1} and G_D^{i-1} such that G_C^{i-1} belongs to G_A^i and G_D^{i-1} belongs to G_B^i and G_C^{i-1} is adjacent to G_D^{i-1}. (From Property 4, G_C^{i-1} is adjacent to G_A^{i-1}, and G_D^{i-1} is adjacent to G_B^{i-1}.)



In this case, $d(A,B) \leq d(A,C) + d(C,D) + d(D,A)$. From the inductive assumption $d(A,C), d(C,D), d(D,A) \leq 3^{i-1}$, thus $d(A,B) \leq 3 \cdot 3^{i-1} = 3^i$.

Consequently, we have $d(A,B) \leq 3^i$. Because G_A^i and G_B^i were chosen arbitrarily, the lemma is true for $i = k+1$.

By applying mathematical induction to the basis and the inductive step, we proved the lemma for all i. Moreover, 3^i is a *tight bound*, that is, it is reachable for some configurations. □

Appendix C
Proof of Lemma 3

Lemma 3: The distance between any two members of a level-i group is at most $3^i - 1$ hops.

Proof: We choose an arbitrary group G_A^i and two arbitrary nodes P and Q that are members of this group. We add another node R to the system such that R's only neighbor is Q.[1] R forms singleton groups $G_R^0 \ldots G_R^i$. From Lemma 1, P can reach R in at most 3^i hops. Since P can reach R only through Q, P can reach Q in at most $3^i - 1$ hops, that is the distance between P and Q is at most $3^i - 1$.

Because P and Q were chosen arbitrarily, the lemma holds for any members of group G_A^i. Likewise, the arbitrary choice of G_A^i and i proves the lemma for all i. Again, $3^i - 1$ is a tight bound. □

Appendix D
Proof of Lemma 4

Lemma 4: Update propagation based on the responsibility rule and update vectors guarantees eventual consistency of node labels: in the absence of changes in the system, for any group G_X^i and any node A, if A is eventually a member of G_X^i (eventually $L(A)[i] = X$), then A and X eventually have equal labels starting from position i (for all $k \geq i$, $L(A)[k] = L(X)[k]$).

Proof: Let $\langle L(A)[i]\rangle_r$ denote the i-th element of the label of node A in round r. Moreover, we treat the first (starting from position 0) *null* value in the label as the end of the label. We observe that, from Property 1, at level 0 every node is always a member of its own singleton group. Therefore, in the remainder of the proof, when referring to eventual consistency at level i, we always assume that $i > 0$.

After changes in the system have stopped, node A eventually becomes a member of G_X^i $(i > 0)$ iff:

$$\exists_{r_A^{s_i}}\ \forall_{r \geq r_A^{s_i}}\ \Big(\big(\langle L(A)[i]\rangle_r = \langle L(X)[i]\rangle_r = X\big)\ \wedge\ \forall_{0\leq j<i}\ \big(\langle U(A)[j]\rangle_r = \langle U(A)[j]\rangle_{r_A^{s_i}}\big)\ \wedge\ \forall_{0\leq j<i}\ \big(\langle L(A)[j]\rangle_r = \langle L(A)[j]\rangle_{r_A^{s_i}} \neq null\big)\Big).$$

In other words there exists a stabilization round for A's label at level i, $r_A^{s_i}$, in and after which: A's label length is greater than i, the i-th element of A's label is always equal to X, and A's label and update vector do not change at lower levels.

All nodes that are eventually members of G_X^i constitute the following set: $\{A \mid A\ \textit{is eventually a member of}\ G_X^i\}$. Since this set is finite, we can define its stabilization round, r^{s_i}, as the maximum stabilization round over all its members. In other words, r^{s_i} denotes the round after which no nodes join or leave group G_X^i. We will use $\langle G_X^i\rangle_{r^{s_i}}$ to denote the stable set of nodes constituting group G_X^i.

We will show that if a node belongs to $\langle G_X^i\rangle_{r^{s_i}}$ its label and X's label will eventually be equal at position i+1. In other words, the two nodes from the same level-i group will eventually belong to the same level-i+1 group. This is sufficient to prove eventual consistency. More formally, for an arbitrary node $A \in \langle G_X^i\rangle_{r^{s_i}}$, we will show that $(***)$:

$$\exists_{r_A^{s_{i+1}}}\ \forall_{r \geq r_A^{s_{i+1}}}\ \Big(\big(\langle L(A)[i+1]\rangle_r = \langle L(X)[i+1]\rangle_r\big)\ \wedge\ \forall_{0\leq j<i+1}\ \big(\langle U(A)[j]\rangle_r = \langle U(A)[j]\rangle_{r_A^{s_{i+1}}}\big)\ \wedge\ \forall_{0\leq j<i+1}\ \big(\langle L(A)[j]\rangle_r = \langle L(A)[j]\rangle_{r_A^{s_{i+1}}} \neq null\big)\Big).$$

1. Although in a practical setting this may be impossible, it is perfectly valid from the graph theory perspective, and consequently, does not invalidate the proof.

Recall the responsibility rule and assume that, before the changes have stopped, the last update by X, acting as a level-i head, was performed in round $r_X^{u_{i+1}}$. The update had the sequence number Δ and corresponded to writing $\bigcirc$ at the i+1-st position of X's label. More formally, the following conditions hold for X's label and update vector:

$$\forall_{r \geq r_X^{u_{i+1}}} \Big(\big(\langle L(X)[i+1]\rangle_r = \bigcirc\big) \;\; \wedge \;\; \big(\langle U(X)[i]\rangle_r = \Delta\big) \Big)$$

We will consider rounds $\geq max(r^{s_i}, r_X^{u_{i+1}})$. We will show that, for an arbitrary node $A \in \langle G_X^i \rangle_{r^{s_i}}$, the following condition holds (‡‡‡):

$$\exists_{r_A^{s_{i+1}} \geq max(r^{s_i}, r_X^{u_{i+1}})} \forall_{r \geq r_A^{s_{i+1}}} \Big(\big(\langle L(A)[i+1]\rangle_r = \bigcirc\big) \;\; \wedge \;\; \big(\langle U(A)[i]\rangle_r = \Delta\big) \Big),$$

which is sufficient to prove $(***)$.

Let us observe that since the group membership is based on connectivity, all members of $\langle G_X^i \rangle_{r^{s_i}}$ form a *connected graph*. We can prove this by induction. For $i = 0$, $\langle G_X^i \rangle_{r^{s_i}} = \{X\}$, hence $\langle G_X^i \rangle_{r^{s_i}}$ is connected as a singleton graph. For $i > 0$, our inductive assumption is that the connectivity property holds for all levels $j < i$. Let us then consider two arbitrary nodes: $P, Q \in \langle G_X^i \rangle_{r^{s_i}}$. Like in earlier proofs, we can have three possibilities. *1):* $P, Q \in \langle G_Y^{i-1} \rangle_{r^{s_i}}$ for some Y, in which case P and Q are connected from the inductive assumption. *2):* $P \in \langle G_Y^{i-1} \rangle_{r^{s_i}}$ and $Q \in \langle G_X^{i-1} \rangle_{r^{s_i}}$ (or vice versa) for some Y. In this case, from the inductive assumption, all nodes in $\langle G_Y^{i-1} \rangle_{r^{s_i}}$ and all nodes in $\langle G_X^{i-1} \rangle_{r^{s_i}}$ are connected. Moreover, from Property 4, since $\langle G_X^{i-1} \rangle_{r^{s_i}}$ is the central subgroup of $\langle G_X^i \rangle_{r^{s_i}}$, it is adjacent to $\langle G_Y^{i-1} \rangle_{r^{s_i}}$, that is, there exists at least one link between $\langle G_X^{i-1} \rangle_{r^{s_i}}$ and $\langle G_Y^{i-1} \rangle_{r^{s_i}}$. Therefore, all nodes from $\langle G_X^{i-1} \rangle_{r^{s_i}}$ (in particular Q) are connected with all nodes from $\langle G_Y^{i-1} \rangle_{r^{s_i}}$ (in particular P) and vice versa. *3):* $P \in \langle G_Y^{i-1} \rangle_{r^{s_i}}$ and $Q \in \langle G_Z^{i-1} \rangle_{r^{s_i}}$ for some Y and Z. Like in *2)*, in this case all nodes from $\langle G_Y^{i-1} \rangle_{r^{s_i}}$ are connected with all nodes from $\langle G_Z^{i-1} \rangle_{r^{s_i}}$ through the central subgroup $\langle G_X^{i-1} \rangle_{r^{s_i}}$. Consequently, P and Q are connected. Since we chose P and Q arbitrarily, the connectivity condition is true for all nodes that belong to $\langle G_X^i \rangle_{r^{s_i}}$. By applying mathematical induction we proved that nodes in any $\langle G_X^i \rangle_{r^{s_i}}$ form a connected graph.

Consequently, we can prove (‡‡‡) by induction over the number of hops, n, from X.

Basis: $n = 0$. The only node, $A \in \langle G_X^i \rangle_{r^{s_i}}$, that is only zero hops away from X is $A = X$, so so $r_A^{s_{i+1}} = max(r^{s_i}, r_X^{u_{i+1}})$. Consequently, (‡‡‡) holds.

Inductive step: $n = k+1$ (where $k \geq 0$). We assume that (‡‡‡) holds for all nodes in $\langle G_X^i \rangle_{r^{s_i}}$ that are $\leq k$ hops away from X. Let us take an arbitrary node, $A \in \langle G_X^i \rangle_{r^{s_i}}$, that is $k+1$ hops away from X. A has a neighbor, $B \in \langle G_X^i \rangle_{r^{s_i}}$, which is k hops away from X and for which (‡‡‡) holds:

$$\exists_{r_B^{s_{i+1}} \geq max(r^{s_i}, r_X^{u_{i+1}})} \forall_{r \geq r_B^{s_{i+1}}} \Big(\big(\langle L(B)[i+1]\rangle_r = \bigcirc\big) \;\; \wedge \;\; \big(\langle U(B)[i]\rangle_r = \Delta\big) \Big).$$

Assume that there is no message loss. In round $r_B^{s_{i+1}} + 1$, A receives a gossip message from B and compares its label against B's label to find the minimal common level (see Sect. 4.3.2). Since A and B are members of $\langle G_X^i \rangle_{r^{s_i}}$, their minimal common level, j is $\leq i$. A then compares its update vector with B's update vector starting from position j. Since both A and B belong to $\langle G_X^i \rangle_{r^{s_i}}$, the first position at which their update vectors can differ is i, otherwise one of them would have to change its update vector in the present round at position $< i$, which precludes membership in $\langle G_X^i \rangle_{r^{s_i}}$. If the update vectors do not differ at position i ($\langle U(B)[i]\rangle_{r_B^{s_{i+1}}+1} = \langle U(A)[i]\rangle_{r_B^{s_{i+1}}+1} = \Delta$), then $\langle L(A)[i]\rangle_{r_B^{s_{i+1}}+1} = \bigcirc$ because the responsibility rule ensures the following invariant:

$$\forall_{P,Q,i,r} \Big(\big(\langle L(P)[i]\rangle_r = \langle L(Q)[i]\rangle_r\big) \;\; \wedge \;\; \big(\langle U(P)[i]\rangle_r = \langle U(Q)[i]\rangle_r\big) \;\; \Rightarrow \big(\langle L(P)[i+1]\rangle_r = \langle L(Q)[i+1]\rangle_r\big) \Big).$$

If the update vectors differ at position i, in turn, then $\langle U(A)[i]\rangle_{r_B^{s_{i+1}}+1} < \Delta$ as Δ is the last update performed by X acting as a level-i head. Consequently, following our consistency enforcement algorithm A copies B's label and update vector starting from position i. As a result, $\langle L(A)[i]\rangle_{r_B^{s_{i+1}}+2} = \bigcirc$ and $\langle U(A)[i]\rangle_{r_B^{s_{i+1}}+2} = \Delta$. In both cases, we have shown that:

$$\exists_{r_A^{s_{i+1}} \geq max(r^{s_i}, r_X^{u_{i+1}})} \Big(\big(\langle L(A)[i+1]\rangle_{r_A^{s_{i+1}}} = \bigcirc\big) \;\; \wedge \;\; \big(\langle U(A)[i]\rangle_{r_A^{s_{i+1}}} = \Delta\big) \Big).$$

Thus, to prove (‡‡‡), we still have to show that A's label element at level i+1 and A's update vector element at level i do not change in any round $r \geq r_A^{s_{i+1}}$. To this end, assume the opposite, that is, in some round $r \geq r_A^{s_{i+1}}$, A's label or update vector change at the mentioned levels. The change cannot be a result of a local update by A at some level $< i$ because this would violate $A \in \langle G_X^i \rangle_{r^{s_i}}$. Therefore, the change must be a result of copying the label of some neighbor, C, after a gossip message from the neighbor has been received. Again, the copying cannot be performed starting from any level $< i$, as this would violate $A \in \langle G_X^i \rangle_{r^{s_i}}$. Therefore, the copying occurred starting from level i. This means that $\langle L(A)[i]\rangle_r = \langle L(C)[i]\rangle_r = X$ and $\langle U(A)[i]\rangle_r = \Delta < \langle U(C)[i]\rangle_r$. We have a *contradiction*, because Δ was the sequence number of the last update performed by X acting as a level-i head, and thus it must be the case that: $\Delta \geq \langle U(C)[i]\rangle_r$. Therefore, we have proved (‡‡‡). Since A was chosen arbitrarily, the inductive step holds for any node that belongs to $\langle G_X^i \rangle_{r^{s_i}}$ and is $k+1$ hops away from X.

By applying mathematical induction to the basis and the inductive step, we proved (‡‡‡) for all n, that is, for all nodes that constitute $\langle G_X^i \rangle_{r^{s_i}}$. Since for any $A \in \langle G_X^i \rangle_{r^{s_i}}$, $(***)$ is a direct consequence of (‡‡‡), we proved $(***)$ for all nodes that belong to $\langle G_X^i \rangle_{r^{s_i}}$. In other words, we proved that, for any level i, any two nodes that eventually have equal labels at level i, also eventually have their labels equal at level i+1.

Because the number of levels is finite, the proof implies that, for any level i, any two nodes that have their label equal at level i, also have their labels equal at all levels $k > i$. This ends the proof of Lemma 4. □

APPENDIX E
PROOF OF LEMMA 5

Lemma 5: Assume that the slot size is longer than the number of rounds it takes to propagate information between the heads, X and Y, of two adjacent "top-level" groups G_X^i and G_Y^i. In this case, with probability $\geq \frac{1}{4}$, G_X^i will be able to join G_Y^{i+1} or vice versa.

Proof: Consider two arbitrary nodes, X and Y, that are heads of adjacent "top-level" groups G_X^i and G_Y^i respectively. X and Y must potentially spawn level-i+1 groups.

Let r_X and r_Y denote the round in which X and Y respectively choose their virtual slots, as described in Sect. 4.4.1. Note that this implies that in round r_X, X has learned about Y, and similarly, in round r_Y, Y has learned about X. Let the number of slots $S = 2$. Moreover, assume that the slot size, R, meets the requirements of the lemma, that is, it is longer than the number of rounds necessary to propagate information between X and Y.

We have the following four possible slot selection configurations, each obtained with probability $\frac{1}{4}$:

	I	II	III	IV
s_X	0	0	1	1
s_Y	0	1	0	1

Without the loss of generality assume that $r_X \geq r_Y$, that is, X selects its slot in the same or later round than Y. Consider configuration III, in which X selects slot $s_X = 1$ and Y selects slot $s_Y = 0$. Let $r_X^* = r_X + s_X \cdot R + 1$ denote the round in which X potentially spawns group G_X^{i+1}, as specified by the algorithm. Likewise, let $r_Y^* = r_Y + s_Y \cdot R + 1$ denote the round in which Y potentially spawns group G_Y^{i+1}. We will show (by contradiction) that by the time it spawns group G_X^{i+1}, X discovers that Y has spawned G_Y^{i+1}. Consequently, X can make G_X^i a subgroup of G_Y^{i+1}, decreasing the number of groups at level i+1.

To this end, assume that X spawns G_X^{i+1} in round r_X^* and Y spawns G_Y^{i+1} in round r_Y^*. Consider value $r_X^* - r_Y^*$ which denotes how many rounds after Y has spawned group G_Y^{i+1}, node X spawns group G_X^{i+1}.

$$\begin{aligned}
r_X^* - r_Y^* &= \\
&= (r_X + s_X \cdot R + 1) - (r_Y + s_Y \cdot R + 1) = \\
&= r_X - r_Y + (s_X - s_Y) \cdot R = \\
&= r_X - r_Y + (1 - 0) \cdot R = \\
&= r_X - r_Y + R \geq^{(from:\ r_X \geq r_Y)} \\
&\geq r_X - r_X + R = \\
&= R.
\end{aligned}$$

From the above calculation X spawns group G_X^{i+1} at least R rounds after Y has spawned group G_Y^{i+1}. **Contradiction!**, because within at most R rounds, X would have learned that Y spawned G_Y^{i+1}, and consequently, would have made G_X^i a subgroup of G_Y^{i+1}. Therefore, with probability at least $\frac{1}{4}$, G_X^i and G_Y^i will be subgroups of a common group $G_{X/Y}^{i+1}$. Because X, Y, G_X^i, and G_Y^i were chosen arbitrarily, Lemma 5 holds for any head node at any level. In practice, the aforementioned probability is higher than $\frac{1}{4}$. □

APPENDIX F
THE MAINTENANCE ALGORITHM

Each node running PL-GOSSIP reacts to two types of events: reception of a gossip message and periodical timeouts. Below, we describe these events in detail. We present the simplest version of the algorithm, without any optimizations.

F.1 Gossip Message Reception

Receiving a message (see Listing 1) allows a node to discover changes in the hierarchy and to update its routing table. A gossip message contains the label of the sender node with the corresponding update vector and the sender's routing table, plus some possible additional information piggybacked by lower layers (e.g., link quality information of the sender's neighbors). First, the node that received the message searches for the minimal common-level group it shares with the sender of the message (listing lines 3-7, see also Sect. 4.3.2). If such a group exists (ll. 9), the node compares its update vector with the sender's update vector to determine which of the two labels is more fresh (ll. 13-17, see also Sect. 4.3.2). If both the labels are fresh (ll. 19), the node only updates its routing table with the entries contained in the gossip message (ll. 20-21). If, however, the sender's label is more fresh (ll. 22), before updating its routing table (ll. 27-28), the node adopts that label as explained in Sect. 4.3.2 (ll. 23-26). Finally, if the sender's label is stale, the node can still use parts of the sender's routing table to update its own routing table (ll. 30-32).

If the node and the sender of the gossip message do not share any group (ll. 35), the node has just discovered a violation of Property 2 (see Sect. 4.4.1). To propagate the information about this violation to the head of its top-level group, the node adds appropriate entries to its routing table, as explained in Sect. 4.2 (ll. 36-43). These entries will allow the head to react to the violation.

F.2 Periodic Timeout

The timeout event (see Listing 2) gives a node the opportunity to react to the changes in the system that occurred since the last timeout. First, the node removes stale entries from its routing table (listing lines 49-50), which enables detecting disruptive failures. More specifically, if the node, being a level-i head (where $i \geq 0$), is not the top-level head (ll. 55), it must check whether the central subgroup of its level-i+1 group is still reachable and adjacent to the node's level-i group (ll. 56-61), as explained in Sect. 4.4.2. If these conditions are not met (a violation of Property 4 occurred), the node cuts its label down to level i (ll. 62-65), as described in Sect. 4.4.2. Otherwise, from the node's perspective, there were no disruptive failures in the system.

Second, if the node is the top-level head (ll. 72, possibly as a result of an earlier label cut), it must check whether the hierarchy construction is complete. To this end, the node first determines if its routing table contains entries for a level-i+1 group it could join (ll. 73-76), as explained in Sect. 4.4.1. If this is the case (ll. 76), the node joins its level-i group to the level-i+1 group, by extending its label with the identifier of

```
1   HANDLER onGossipReceived(msg) {
2
3     // determine if we share any group
4     int i = 0;
5     for (; i < min(this.lab.len, msg.lab.len); ++i)
6       if (this.lab[i] == msg.lab[i])
7         break;
8
9     if (i < min(this.lab.len, msg.lab.len)) {
10      // we found a node that shares a group with us,
11      // so determine who has a more recent label
12
13      // find the minimal differing position
14      int j = i;
15      for (; j < min(this.lab.len, msg.lab.len); ++j)
16        if (this.uvec[j] != msg.uvec[j])
17          break;
18
19      if (j >= min(this.lab.len, msg.lab.len)) {
20        // we both have the same labels
21        this.rt.mergeWith(msg.rt, i − 1, msg.rt.topRow);
22      } else if (this.uvec[j] < msg.uvec[j]) {
23        // we are not up to date, so
24        // change our label and update vector
25        this.lab.copyFrom(msg.lab, j);
26        this.uvec.copyFrom(msg.uvec, j);
27        // merge routing tables
28        this.rt.mergeWith(msg.rt, i − 1, msg.rt.topRow);
29      } else {
30        // the other guy is not up to date, but we
31        // can still use a part of his routing table
32        this.rt.mergeWith(msg.rt, i − 1, j);
33      }
34
35    } else {
36      // we encountered a node from a completely
37      // different group (a violation of Property 2),
38      // so add the groups of the encountered node
39      if (msg.lab.len >= this.lab.len)
40        this.rt.mergeWithNodeGroups(
41            msg.lab, msg.rt,
42            this.lab.len − 1,
43            msg.lab.len − 1);
44    }
45  }
```

Listing 1. The handler of the gossip message reception.

the head of this level-i+1 group (ll. 77-81). It also cancels any possible pending suppression of label extension which corresponded to spawning a new supergroup (ll. 82-83). As explained in Sect. 4.4.1, if the level-i+1 group is itself a member of some higher-level groups, all members of the node's level-i group will gradually extend their labels when exchanging gossip messages (ll. 22-28).

Even if an appropriate level-i+1 group to join could not be found, it is still possible that the hierarchy is not complete. More specifically, the node must check whether its routing table contains any entries for other groups starting from level i (ll. 88). If so the node activates a suppression counter to defer spawning a new level-$i+1$ group (ll. 99-103), as explained in Sect. 4.4.1. The suppression counter, once activated, is decremented during each timeout (ll. 107-108). When it reaches zero and the level-i+1 group still has to be spawned (ll. 90), the node extends its label and cancels the counter (ll. 91-97), effectively spawning a new level-i+1 group (with itself as the head of that group).

Finally, when the node reacted to all changes in the system, it broadcasts a gossip message (ll. 117-118), such that its neighbors can adopt any label updates and update the routes.

```
47  HANDLER onTimeout() {
48    int olducnt = this.ucnt;
49
50    // evict dead entries from the routing table
51    this.rt.ageAndClean();
52
53    int i = this.lab.getHeadLevel();
54
55    // check if we need to cut the label
56    if (i + 1 < this.lab.len) {
57      // we are not the top level head, so check if
58      // our superhead died or ceased to be adjacent
59      RtEntry centralSubgroupEntry =
60        this.rt[i][this.lab[i + 1]];
61      if (centralSubgroupEntry == null
62              || !centralSubgroupEntry.isAdjacent) {
63        // perform the label cut operation
64        this.lab.cutTo(i);
65        this.uvec.cutTo(i);
66        this.uvec[i] = ++this.ucnt;
67      } else {
68        // our superhead works so there is nothing to do
69      }
70    }
71
72    // check if we need to extend the label
73    if (i + 1 == this.lab.len) {
74      // we are the top level head, so check if there is
75      // any same− or higher−level group we could join
76      JoinCandidate jc = this.rt.getJoinCandidate();
77      if (jc != null) {
78        // we have a group which we can join,
79        // so perform the label extension operation
80        this.lab.extendWith(jc.group);
81        this.uvec.extendWith(0);
82        this.uvec[i] = ++this.ucnt;
83        // reset suppression counter
84        this.scnt = −1;
85      } else {
86        // we do not have such a group
87        if (this.scnt <= 0) {
88          // check if we need to extend the label
89          if (this.rt.hasOtherEntriesUpFrom(i)) {
90            // yes, we do have to extend the label...
91            if (this.scnt == 0) {
92              // our surrpression timer just fired,
93              // so perform the label extension
94              this.lab.extendWith(this.lab[0]);
95              this.uvec.extendWith(0);
96              this.uvec[i] = ++this.ucnt;
97              // reset surrpression counter
98              this.scnt = −1;
99            } else {
100             // we have to activate the counter
101             this.scnt = selectRandSlot(i) ∗
102                 normalize(
103                   min(intpow(3, i), MAX_PATH),
104                   θ);
105           }
106         }
107       } else {
108         // our surrpression timer is ticking
109         −−this.scnt;
110       }
111     }
112   }
113
114
115   if (olducnt < this.ucnt)
116     save("UPDATE_CNT", this.ucnt);
117
118   // broadcast the gossip message
119   broadcastGossip(this.lab, this.uvec, this.rt);
120 }
```

Listing 2. The periodical timer handler.

```
122 HANDLER onNodeBoot() {
123   // initialize
124   this.lab = {this.NODE_ID};
125   this.uvec = {0};
126   this.rt = {}{};
127   this.scnt = −1;
128   this.ucnt = restore("UPDATE_CNT");
129
130   // set timer handler
131   setTimer(ΔT, &onTimeout);
132 }
```

Listing 3. The initialization handler.

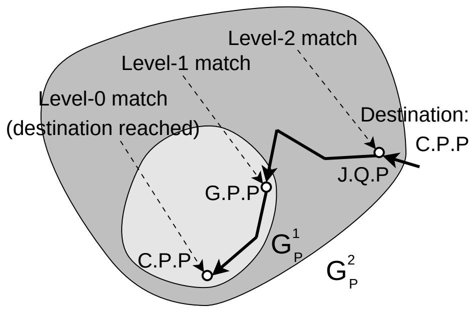


Fig. 1. An example of hierarchical routing in the hierarchy from Fig. 1 of the paper.

F.3 Remarks

When a node repaired after a failure rejoins the system, its membership decisions (label updates) made before the failure may still be present in the labels of other nodes. Therefore, it is crucial to ensure that any decision made by this node after the failure is perceived by other nodes as later than any decision made by this node before the failure. Otherwise, the ordering of label updates is not preserved, which disrupts the consistency enforcement algorithm. In that case, we cannot predict the behavior of the system.

To this end, whenever a node performs a label update it stores the new value of the update counter persistently, for instance, in the local flash memory (ll. 114-115). During boot, the node restores the last value of the counter from the persistent storage (see Listing 3, line 126), which ensures correct ordering of any subsequent membership decisions. Alternatively, a node rejoining the system obtains a new unique identifier which eliminates the problem completely.

Appendix G
Hierarchical Suffix-Based Routing

Routing is performed by resolving consecutive elements of the destination label starting from the maximal-position element differing at the sender (see Fig. 1). The main routing method, executed by a node on each hop, is presented in Listing 4.

Upon reception of an *application* message (which is different from a gossip message used by PL-Gossip to maintain the network structure), a node decrements the *time-to-live* (TTL) counter associated with the message and examines this counter to decide whether the message should be dropped (listing line 2-6). TTL is a mechanism for dropping messages that cannot be delivered to their receivers due to network dynamics

```
1  FUNCTION getNextHop(msg) {
2
3    // change TTL of the message
4    −−msg.ttl;
5    if (msg.ttl < 0)
6      return null;
7
8    // determine if we share any group
9    int cpos = 0;
10   for (; cpos < min(this.lab.len, msg.dstLab.len); ++cpos)
11     if (this.lab[cpos] == msg.dstLab[cpos])
12       break;
13
14   if (cpos == 0} {
15     // we are the destination node
16     acceptMessage(msg);
17     return null;
18
19   } else if (this.neighbors.contains(msg.dstLab[0])) {
20     // one of our neighbors is the destination node
21     // (this is just an optimization)
22     return msg.dstLab[0];
23
24   } else if (cpos <= min(this.lab.len, msg.dstLab.len)) {
25     // resolve the next hop based on the routing table
26     Entry entry = this.rt[cpos − 1][msg.dstLab[cpos − 1]];
27     return entry != null ? entry.nextHop : null;
28
29   } else {
30     // we cannot forward the message
31     return null;
32   }
33 }
```

Listing 4. The main routing function.

```
35 FUNCTION initMessage(dstLab, data) {
36
37   // create a new message
38   Message msg = new Message();
39   msg.dstLab = dstLab;
40   msg.data = data;
41
42   // compute TTL based on Lemma 3
43   int i;
44   for (i = 0; i < min(dstLab.len, this.lab.len); ++i) {
45     if (dstLab[i] == this.lab[i])
46       break;
47   }
48   msg.ttl = min(intpow(3, i) − 1, MAX_PATH);
49 }
```

Listing 5. The message initialization function.

(e.g., receiver failures). The TTL counter of a message is set by the originator of this message based on Lemma 3 (see Listing 5). More specifically, the originator resolves the minimal-level group it shares with the destination node (ll. 41-46;), suppose the level of this group is i, and sets the TTL counter accordingly to $3^i - 1$.

If the message has not been dropped, the node determines how many elements of the destination label are left to be resolved (ll. 8-12). If there are no such elements left, then the present node is the destination and thus, it accepts the message (ll. 14-17). Otherwise, the message must be forwarded. As an optimization, the node first checks whether one of its neighbors is the destination node and if so, it forwards the message to this neighbor (ll. 19-22). If there are no such neighbors, the next hop is determined based on the routing table. More specifically, the present nodes looks up an entry for the next unresolved element of the destination, and forwards the message to the next hop neighbor associated with this entry (ll. 24-37). Finally, it may happen that due to hierarchy disturbance, the next hop cannot be resolved. In this case, the node drops the message (the main routing method returns *null*).