Distributed Systems

(3rd Edition)

Chapter 06: Coordination

Version: February 25, 2017

Physical clocks

Problem

Sometimes we simply need the exact time, not just an ordering.

Solution: Universal Coordinated Time (UTC)

- Based on the number of transitions per second of the cesium 133 atom (pretty accurate).
- At present, the real time is taken as the average of some 50 cesium clocks around the world.
- Introduces a leap second from time to time to compensate that days are getting longer.

Note

UTC is broadcast through short-wave radio and satellite. Satellites can give an accuracy of about ± 0.5 ms.

Clock synchronization

Precision

The goal is to keep the deviation between two clocks on any two machines within a specified bound, known as the precision π:

$$\forall t, \forall p, q : |C_p(t) - C_q(t)| \leq \pi$$

with $C_p(t)$ the computed clock time of machine p at UTC time t.

Accuracy

In the case of accuracy, we aim to keep the clock bound to a value α:

$$\forall t, \forall p : |C_p(t) - t| \leq \alpha$$

Synchronization

- Internal synchronization: keep clocks precise
- External synchronization: keep clocks accurate

Clock drift

Clock specifications

- A clock comes specified with its maximum clock drift rate ρ.
- $F(t)$ denotes oscillator frequency of the hardware clock at time t
- F is the clock's ideal (constant) frequency $\Rightarrow$ living up to specifications:

$$\forall t : (1-\rho) \leq \frac{F(t)}{F} \leq (1+\rho)$$

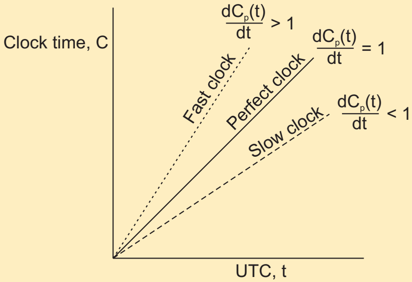
Observation

By using hardware interrupts we couple a software clock to the hardware clock, and thus also its clock drift rate:

$$C_p(t) = \frac{1}{F}\int_0^t F(t)dt \Rightarrow \frac{dC_p(t)}{dt} = \frac{F(t)}{F}$$

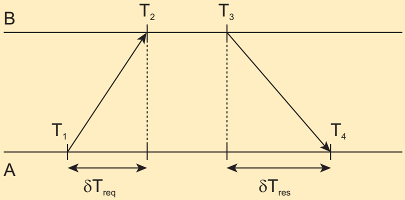
$$\Rightarrow \forall t : 1-\rho \leq \frac{dC_p(t)}{dt} \leq 1+\rho$$

Fast, perfect, slow clocks



Detecting and adjusting incorrect times

Getting the current time from a time server



Computing the relative offset θ and delay δ

Assumption: $\delta T_{req} = T_2 - T_1 \approx T_4 - T_3 = \delta T_{res}$

$$\theta = T_3 + \big((T_2 - T_1) + (T_4 - T_3)\big)/2 - T_4 = \big((T_2 - T_1) + (T_3 - T_4)\big)/2$$

$$\delta = \big((T_4 - T_1) - (T_3 - T_2)\big)/2$$

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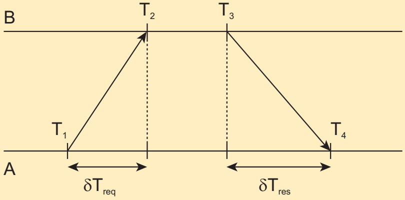

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Network Time Protocol

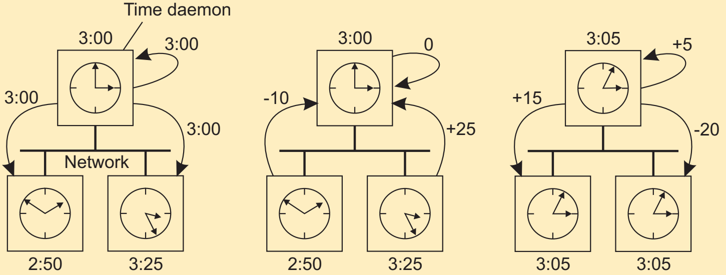
Collect eight (θ, δ) pairs and choose θ for which associated delay δ was minimal.

Keeping time without UTC

Principle

Let the time server scan all machines periodically, calculate an average, and inform each machine how it should adjust its time relative to its present time.

Using a time server



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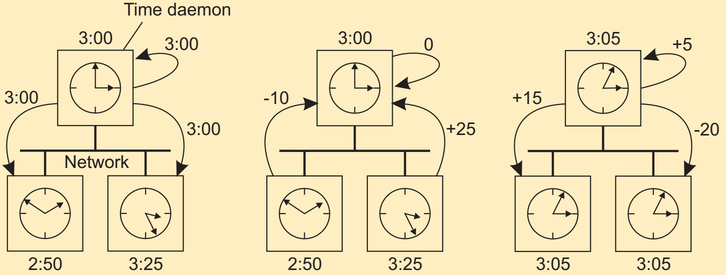


Fundamental

You'll have to take into account that setting the time back is never allowed ⇒ smooth adjustments (i.e., run faster or slower).

Reference broadcast synchronization

Essence

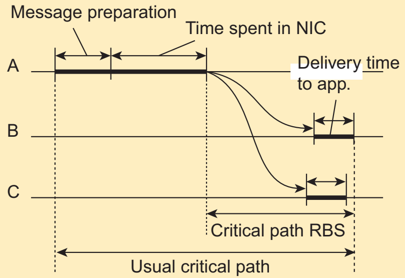
- A node broadcasts a reference message $m \Rightarrow$ each receiving node p records the time $T_{p,m}$ that it received m.
- Note: $T_{p,m}$ is read from p's local clock.

Problem: averaging will not capture drift $\Rightarrow$ use linear regression

NO: $\mathit{Offset}[p,q](t) = \frac{\sum_{k=1}^{M}(T_{p,k} - T_{q,k})}{M}$

YES: $\mathit{Offset}[p,q](t) = \alpha t + \beta$

RBS minimizes critical path



The Happened-before relationship

Issue

What usually matters is not that all processes agree on exactly what time it is, but that they agree on the order in which events occur. Requires a notion of ordering.

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The happened-before relation

- If a and b are two events in the same process, and a comes before b, then $a \to b$.
- If a is the sending of a message, and b is the receipt of that message, then $a \to b$
- If $a \to b$ and $b \to c$, then $a \to c$

Note

This introduces a partial ordering of events in a system with concurrently operating processes.

Logical clocks

Problem

How do we maintain a global view on the system's behavior that is consistent with the happened-before relation?

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Attach a timestamp $C(e)$ to each event e, satisfying the following properties:

- P1 If a and b are two events in the same process, and $a \rightarrow b$, then we demand that $C(a) < C(b)$.
- P2 If a corresponds to sending a message m, and b to the receipt of that message, then also $C(a) < C(b)$.

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Problem

How to attach a timestamp to an event when there's no global clock $\Rightarrow$ maintain a consistent set of logical clocks, one per process.

Logical clocks: solution

Each process P_i maintains a local counter C_i and adjusts this counter

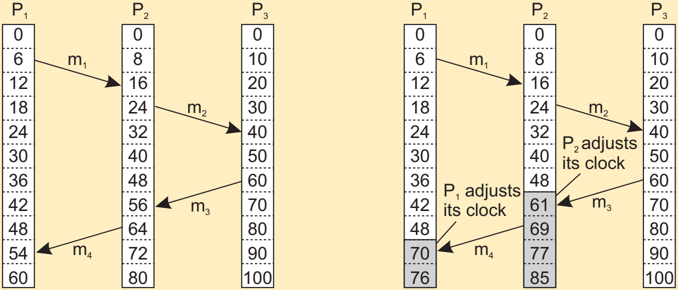
1. For each new event that takes place within P_i, C_i is incremented by 1.
2. Each time a message m is sent by process P_i, the message receives a timestamp $ts(m) = C_i$.
3. Whenever a message m is received by a process P_j, P_j adjusts its local counter C_j to $\max\{C_j, ts(m)\}$; then executes step 1 before passing m to the application.

Notes

- Property P1 is satisfied by (1); Property P2 by (2) and (3).
- It can still occur that two events happen at the same time. Avoid this by breaking ties through process IDs.

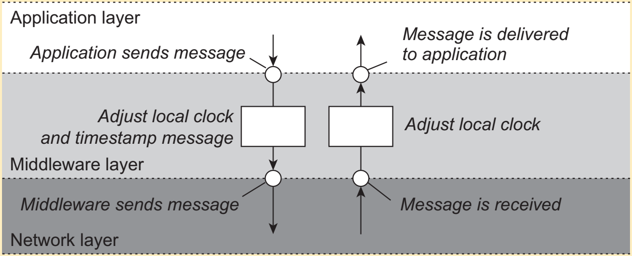
Logical clocks: example

Consider three processes with event counters operating at different rates



Logical clocks: where implemented

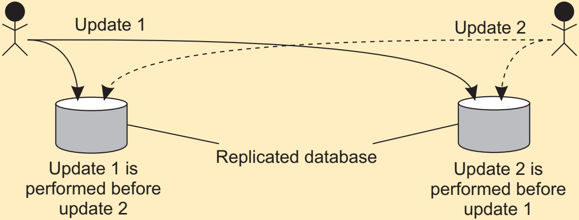
Adjustments implemented in middleware



Example: Total-ordered multicast

Concurrent updates on a replicated database are seen in the same order everywhere

- P_1 adds \$100 to an account (initial value: \$1000)
- P_2 increments account by 1%
- There are two replicas



Result

In absence of proper synchronization:
replica #1 $\leftarrow$ \$1111, while replica #2 $\leftarrow$ \$1110.

Example: Total-ordered multicast

Solution

- Process P_i sends timestamped message m_i to all others. The message itself is put in a local queue $queue_i$.
- Any incoming message at P_j is queued in $queue_j$, according to its timestamp, and acknowledged to every other process.

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P_j passes a message m_i to its application if:

(1) m_i is at the head of $queue_j$

(2) for each process P_k, there is a message m_k in $queue_j$ with a larger timestamp.

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Note

We are assuming that communication is reliable and FIFO ordered.

Lamport's clocks for mutual exclusion

```
class Process:
  def __init__(self, chan):
    self.queue      = []                       # The request queue
    self.clock      = 0                        # The current logical clock

  def requestToEnter(self):
    self.clock = self.clock + 1                          # Increment clock value
    self.queue.append((self.clock, self.procID, ENTER)) # Append request to q
    self.cleanupQ()                                      # Sort the queue
    self.chan.sendTo(self.otherProcs, (self.clock,self.procID,ENTER)) # Send request

  def allowToEnter(self, requester):
    self.clock = self.clock + 1                          # Increment clock value
    self.chan.sendTo([requester], (self.clock,self.procID,ALLOW)) # Permit other

  def release(self):
    tmp = [r for r in self.queue[1:] if r[2] == ENTER]  # Remove all ALLOWs
    self.queue = tmp                                     # and copy to new queue
    self.clock = self.clock + 1                          # Increment clock value
    self.chan.sendTo(self.otherProcs, (self.clock,self.procID,RELEASE)) # Release

  def allowedToEnter(self):
    commProcs = set([req[1] for req in self.queue[1:]]) # See who has sent a message
    return (self.queue[0][1]==self.procID and len(self.otherProcs)==len(commProcs))
```

Lamport's clocks for mutual exclusion

```
def receive(self):
  msg = self.chan.recvFrom(self.otherProcs)[1]        # Pick up any message
  self.clock = max(self.clock, msg[0])                # Adjust clock value...
  self.clock = self.clock + 1                         # ...and increment
  if msg[2] == ENTER:
    self.queue.append(msg)                            # Append an ENTER request
    self.allowToEnter(msg[1])                         # and unconditionally allow
  elif msg[2] == ALLOW:
    self.queue.append(msg)                            # Append an ALLOW
  elif msg[2] == RELEASE:
    del(self.queue[0])                                # Just remove first message
  self.cleanupQ()                                     # And sort and cleanup
```

Lamport's clocks for mutual exclusion

Analogy with total-ordered multicast

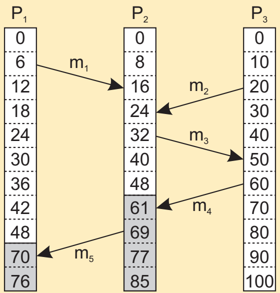
- With total-ordered multicast, all processes build identical queues, delivering messages in the same order
- Mutual exclusion is about agreeing in which order processes are allowed to enter a critical section

Vector clocks

Observation

Lamport's clocks do not guarantee that if $C(a) < C(b)$ that a causally preceded b.

Concurrent message transmission using logical clocks



Observation

Event a: m_1 is received at $T = 16$;
Event b: m_2 is sent at $T = 20$.

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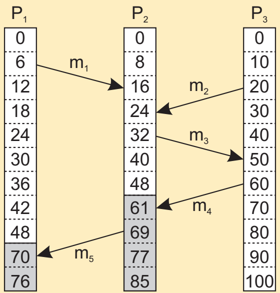


Observation

Event a: m_1 is received at $T = 16$;
Event b: m_2 is sent at $T = 20$.

Note

We cannot conclude that a causally precedes b.

Causal dependency

Definition

We say that *b* may causally depend on *a* if $ts(a) < ts(b)$, with:

- for all k, $ts(a)[k] \leq ts(b)[k]$ and
- there exists at least one index k' for which $ts(a)[k'] < ts(b)[k']$

Precedence vs. dependency

- We say that *a* causally precedes *b*.
- *b* may causally depend on *a*, as there may be information from *a* that is propagated into *b*.

Capturing causality

Solution: each P_i maintains a vector VC_i

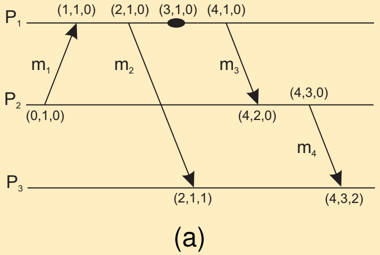
- $VC_i[i]$ is the local logical clock at process P_i.
- If $VC_i[j] = k$ then P_i knows that k events have occurred at P_j.

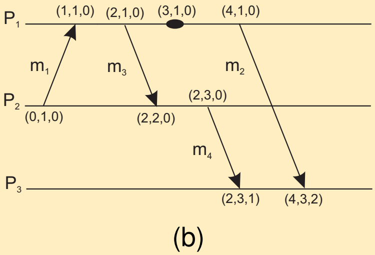
Maintaining vector clocks

1. Before executing an event P_i executes $VC_i[i] \leftarrow VC_i[i] + 1$.
2. When process P_i sends a message m to P_j, it sets m's (vector) timestamp $ts(m)$ equal to VC_i after having executed step 1.
3. Upon the receipt of a message m, process P_j sets $VC_j[k] \leftarrow \max\{VC_j[k], ts(m)[k]\}$ for each k, after which it executes step 1 and then delivers the message to the application.

Vector clocks: Example

Capturing potential causality when exchanging messages





Analysis

Situation	$ts(m_2)$	$ts(m_4)$	$ts(m_2) < ts(m_4)$	$ts(m_2) > ts(m_4)$	**Conclusion**
(a)	$(2,1,0)$	$(4,3,0)$	Yes	No	m_2 may causally precede m_4
(b)	$(4,1,0)$	$(2,3,0)$	No	No	m_2 and m_4 may conflict

Causally ordered multicasting

Observation

We can now ensure that a message is delivered only if all causally preceding messages have already been delivered.

Adjustment

P_i increments $VC_i[i]$ only when sending a message, and P_j "adjusts" VC_j when receiving a message (i.e., effectively does not change $VC_j[j]$).

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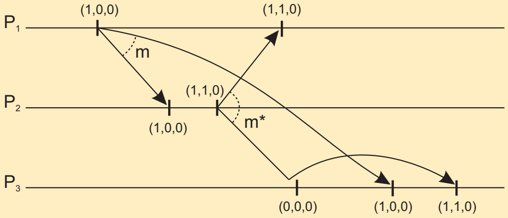
P_j postpones delivery of m until:

1. $ts(m)[i] = VC_j[i] + 1$
2. $ts(m)[k] \leq VC_j[k]$ for all $k \neq i$

Causally ordered multicasting

Enforcing causal communication



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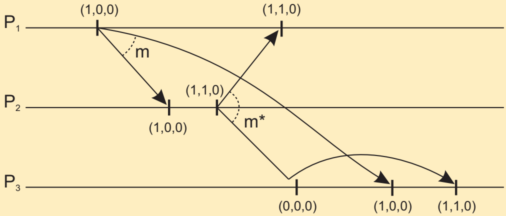


Example

Take $VC_3 = [0,2,2], ts(m) = [1,3,0]$ from P_1. What information does P_3 have, and what will it do when receiving m (from P_1)?

Mutual exclusion

Problem

A number of processes in a distributed system want exclusive access to some resource.

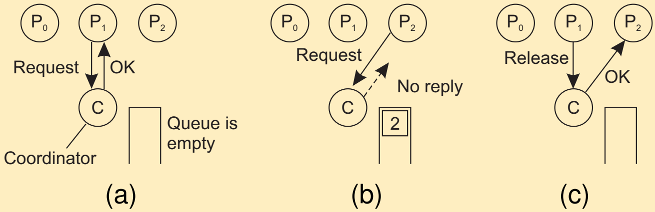
Basic solutions

Permission-based: A process wanting to enter its critical section, or access a resource, needs permission from other processes.

Token-based: A token is passed between processes. The one who has the token may proceed in its critical section, or pass it on when not interested.

Permission-based, centralized

Simply use a coordinator



(a) Process P_1 asks the coordinator for permission to access a shared resource. Permission is granted.

(b) Process P_2 then asks permission to access the same resource. The coordinator does not reply.

(c) When P_1 releases the resource, it tells the coordinator, which then replies to P_2.

Mutual exclusion Ricart & Agrawala

The same as Lamport except that acknowledgments are not sent

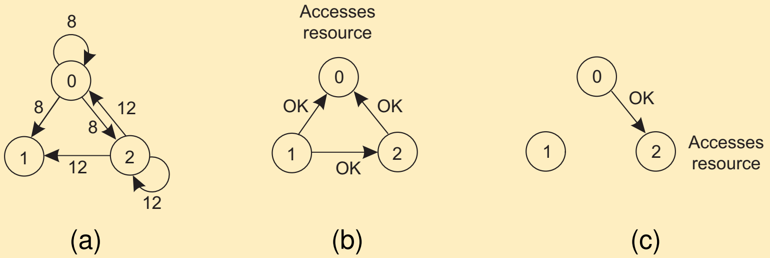
Return a response to a request only when:

- The receiving process has no interest in the shared resource; or
- The receiving process is waiting for the resource, but has lower priority (known through comparison of timestamps).

In all other cases, reply is deferred, implying some more local administration.

Mutual exclusion Ricart & Agrawala

Example with three processes



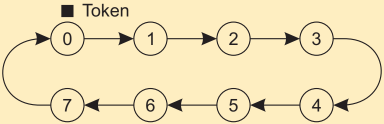
(a) Two processes want to access a shared resource at the same moment.
(b) P_0 has the lowest timestamp, so it wins.
(c) When process P_0 is done, it sends an *OK* also, so P_2 can now go ahead.

Mutual exclusion: Token ring algorithm

Essence

Organize processes in a logical ring, and let a token be passed between them. The one that holds the token is allowed to enter the critical region (if it wants to).

An overlay network constructed as a logical ring with a circulating token



Decentralized mutual exclusion

Principle

Assume every resource is replicated N times, with each replica having its own coordinator $\Rightarrow$ access requires a majority vote from $m > N/2$ coordinators. A coordinator always responds immediately to a request.

Assumption

When a coordinator crashes, it will recover quickly, but will have forgotten about permissions it had granted.

Decentralized mutual exclusion

How robust is this system?

- Let $p = \Delta t / T$ be the probability that a coordinator resets during a time interval Δt, while having a lifetime of T.

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- f coordinators reset $\Rightarrow$ correctness is violated when there is only a minority of nonfaulty coordinators: when $m - f \leq N/2$, or, $f \geq m - N/2$.

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- f coordinators reset $\Rightarrow$ correctness is violated when there is only a minority of nonfaulty coordinators: when $m - f \leq N/2$, or, $f \geq m - N/2$.
- The probability of a violation is $\sum_{k=m-N/2}^{N} \mathbb{P}[k]$.

Decentralized mutual exclusion

Violation probabilities for various parameter values

N	m	p	Violation
8	5	3 sec/hour	$< 10^{-15}$
8	6	3 sec/hour	$< 10^{-18}$
16	9	3 sec/hour	$< 10^{-27}$
16	12	3 sec/hour	$< 10^{-36}$
32	17	3 sec/hour	$< 10^{-52}$
32	24	3 sec/hour	$< 10^{-73}$

N	m	p	Violation
8	5	30 sec/hour	$< 10^{-10}$
8	6	30 sec/hour	$< 10^{-11}$
16	9	30 sec/hour	$< 10^{-18}$
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So....

What can we conclude?

Mutual exclusion: comparison

Algorithm	**Messages per entry/exit**	**Delay before entry (in message times)**
Centralized	3	2
Distributed	$2 \cdot (N-1)$	$2 \cdot (N-1)$
Token ring	$1, \ldots, \infty$	$0, \ldots, N-1$
Decentralized	$2 \cdot m \cdot k + m, k = 1, 2, \ldots$	$2 \cdot m \cdot k$

Election algorithms

Principle

An algorithm requires that some process acts as a coordinator. The question is how to select this special process dynamically.

Note

In many systems the coordinator is chosen by hand (e.g. file servers). This leads to centralized solutions $\Rightarrow$ single point of failure.

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Teasers

1. If a coordinator is chosen dynamically, to what extent can we speak about a centralized or distributed solution?
2. Is a fully distributed solution, i.e. one without a coordinator, always more robust than any centralized/coordinated solution?

Basic assumptions

- All processes have unique id's
- All processes know id's of all processes in the system (but not if they are up or down)
- Election means identifying the process with the highest id that is up

Election by bullying

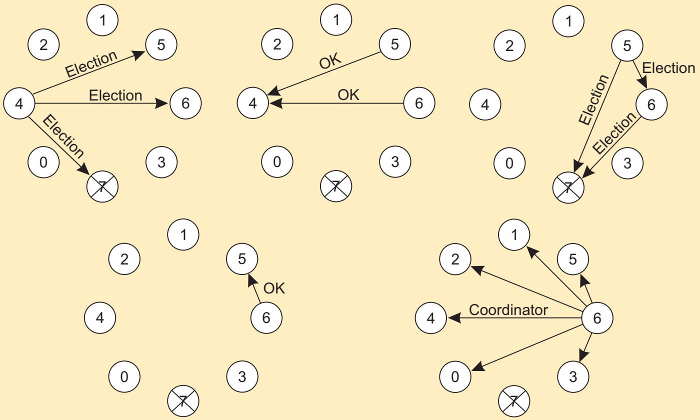
Principle

Consider N processes $\{P_0, \ldots, P_{N-1}\}$ and let $id(P_k) = k$. When a process P_k notices that the coordinator is no longer responding to requests, it initiates an election:

1. P_k sends an *ELECTION* message to all processes with higher identifiers: $P_{k+1}, P_{k+2}, \ldots, P_{N-1}$.
2. If no one responds, P_k wins the election and becomes coordinator.
3. If one of the higher-ups answers, it takes over and P_k's job is done.

Election by bullying

The bully election algorithm



Election in a ring

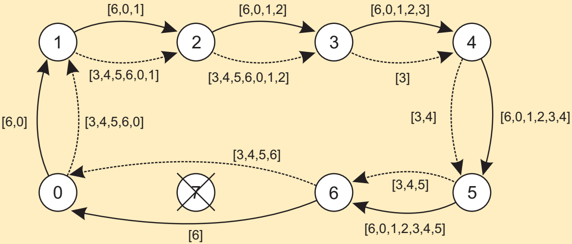
Principle

Process priority is obtained by organizing processes into a (logical) ring. Process with the highest priority should be elected as coordinator.

- Any process can start an election by sending an election message to its successor. If a successor is down, the message is passed on to the next successor.
- If a message is passed on, the sender adds itself to the list. When it gets back to the initiator, everyone had a chance to make its presence known.
- The initiator sends a coordinator message around the ring containing a list of all living processes. The one with the highest priority is elected as coordinator.

Election in a ring

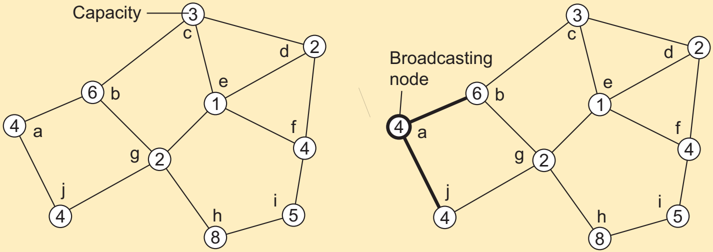
Election algorithm using a ring



- The solid line shows the election messages initiated by P_6
- The dashed one the messages by P_3

A solution for wireless networks

A sample network



A solution for wireless networks

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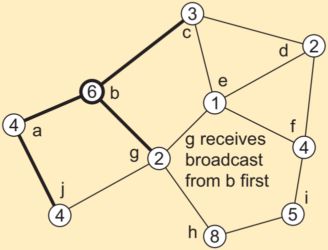

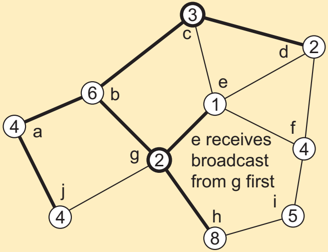

A solution for wireless networks

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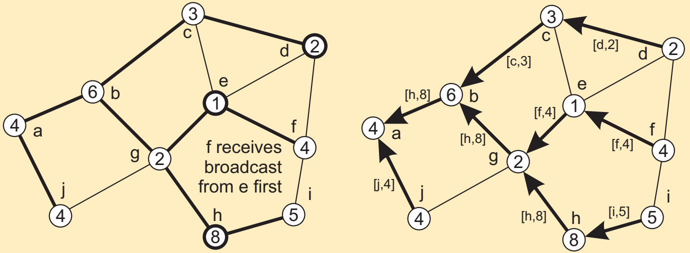


Positioning nodes

Issue

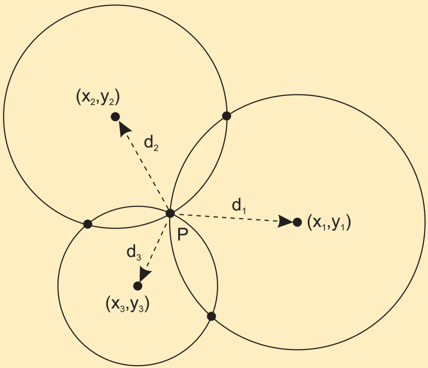
In large-scale distributed systems in which nodes are dispersed across a wide-area network, we often need to take some notion of proximity or distance into account $\Rightarrow$ it starts with determining a (relative) location of a node.

Computing position

Observation

A node P needs $d+1$ landmarks to compute its own position in a d-dimensional space. Consider two-dimensional case.

Computing a position in 2D



Solution

P needs to solve three equations in two unknowns (x_P,y_P):

$$d_i = \sqrt{(x_i - x_P)^2 + (y_i - y_P)^2}$$

Global positioning system

Assuming that the clocks of the satellites are accurate and synchronized

- It takes a while before a signal reaches the receiver
- The receiver's clock is definitely out of sync with the satellite

Basics

Observation

4 satellites $\Rightarrow$ 4 equations in 4 unknowns (with Δ_r as one of them)

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- $\Delta_i = (T_{now} - T_i) + \Delta_r$: measured delay of the message sent by satellite i.
- Measured distance to satellite i: $c \times \Delta_i$ (c is speed of light)
- Real distance: $d_i = c\Delta_i - c\Delta_r = \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}$

Observation

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WiFi-based location services

Basic idea

- Assume we have a database of known access points (APs) with coordinates
- Assume we can estimate distance to an AP
- Then: with 3 detected access points, we can compute a position.

War driving: locating access points

- Use a WiFi-enabled device along with a GPS receiver, and move through an area while recording observed access points.
- Compute the centroid: assume an access point *AP* has been detected at N different locations $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\}$, with known GPS location.
- Compute location of *AP* as $\vec{x}_{AP} = \frac{\sum_{i=1}^{N} \vec{x}_i}{N}$.

Problems

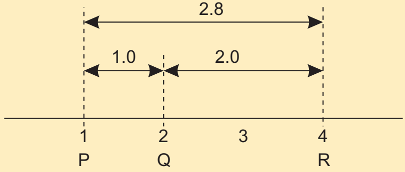
- Limited accuracy of each GPS detection point $\vec{x}_i$
- An access point has a nonuniform transmission range
- Number of sampled detection points N may be too low.

Computing position

Problems

- Measured latencies to landmarks fluctuate
- Computed distances will not even be consistent

Inconsistent distances in 1D space



Solution: minimize errors

- Use N special landmark nodes $L_1, \ldots, L_N$.
- Landmarks measure their pairwise latencies $\tilde{d}(L_i, L_j)$
- A central node computes the coordinates for each landmark, minimizing:

$$\sum_{i=1}^{N} \sum_{j=i+1}^{N} \left(\frac{\tilde{d}(L_i, L_j) - \hat{d}(L_i, L_j)}{\tilde{d}(L_i, L_j)} \right)^2$$

where $\hat{d}(L_i, L_j)$ is distance after nodes L_i and L_j have been positioned.

Computing position

Choosing the dimension m

The hidden parameter is the dimension m with $N > m$. A node P measures its distance to each of the N landmarks and computes its coordinates by minimizing

$$\sum_{i=1}^{N}\left(\frac{\tilde{d}(L_i,P)-\hat{d}(L_i,P)}{\tilde{d}(L_i,P)}\right)^2$$

Observation

Practice shows that m can be as small as 6 or 7 to achieve latency estimations within a factor 2 of the actual value.

Vivaldi

Principle: network of springs exerting forces

Consider a collection of N nodes $P_1, \ldots, P_N$, each P_i having coordinates $\vec{x}_i$. Two nodes exert a mutual force:

$$\vec{F}_{ij} = \left(\tilde{d}(P_i, P_j) - \hat{d}(P_i, P_j)\right) \times u(\vec{x}_i - \vec{x}_j)$$

with $u(\vec{x}_i - \vec{x}_j)$ is the unit vector in the direction of $\vec{x}_i - \vec{x}_j$

Node P_i repeatedly executes steps

1. Measure the latency $\tilde{d}_{ij}$ to node P_j, and also receive P_j's coordinates $\vec{x}_j$.
2. Compute the error $e = \tilde{d}(P_i, P_j) - \hat{d}(P_i, P_j)$
3. Compute the direction $\vec{u} = u(\vec{x}_i - \vec{x}_j)$.
4. Compute the force vector $F_{ij} = e \cdot \vec{u}$
5. Adjust own position by moving along the force vector: $\vec{x}_i \leftarrow \vec{x}_i + \delta \cdot \vec{u}$.

Example applications

Typical apps

- Data dissemination: Perhaps the most important one. Note that there are many variants of dissemination.
- Aggregation: Let every node P_i maintain a variable v_i. When two nodes gossip, they each reset their variable to

$$v_i, v_j \leftarrow (v_i + v_j)/2$$

Result: in the end each node will have computed the average $\bar{v} = \sum_i v_i / N$.

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  Result: in the end each node will have computed the average $\bar{v} = \sum_i v_i/N$.
- What happens in the case that initially $v_i = 1$ and $v_j = 0, j \neq i$?