Graph Theory and Complex Networks: An Introduction

Maarten van Steen

VU Amsterdam, Dept. Computer Science
Room R4.20, steen@cs.vu.nl

Chapter 08: Computer networks

Version: March 3, 2011

vrije Universiteit amsterdam

Contents

2 / 53

Introduction

Observation

The Internet as we know it today is a communication network that allows us to exchange messages. The (World Wide) Web is a huge distributed information system, implemented on top of the Internet. The two are very different.

1. The organization and structure of the Internet
2. The organization of overlay (i.e., peer-to-peer) networks
3. The organization and structure of the Web

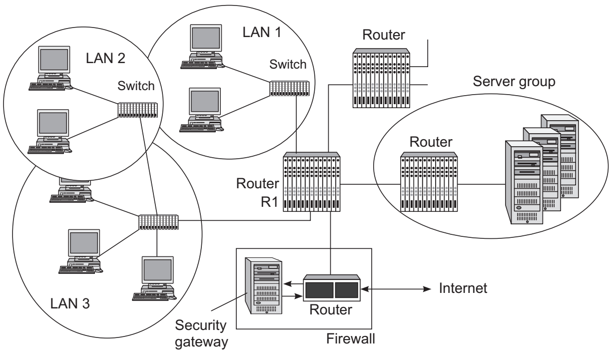
3 / 53

Computer networks: basics

- There are **many** different kinds of computer networks:
  - Traditional networks in buildings and on campus
  - Home networks (wired and wireless)
  - Networks for mobile phones
  - Access networks (with so-called hot spots)
  - Networks owned by Internet Service Providers (ISPs)
  - ...
- The Internet ties all these networks together (well, that's what we think).
- For starters: make distinction between small-area networks and large-area networks.

4 / 53

Small-area networks



5 / 53

Example: router

6 / 53

Example: switch

Example: security gateway

Example: server group

Addressing

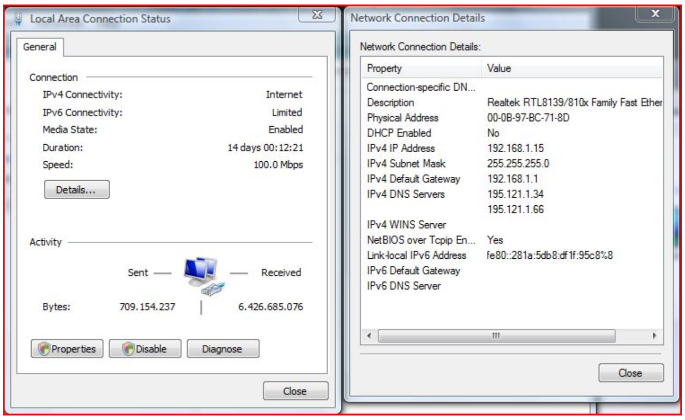
Essence

Each networked device has (in principle) a worldwide unique **low-level address**, also called its **MAC address**. The MAC address is nothing but a **device identifier**.

- When a device transmits a message, it always sends its MAC address as part of the message.
- A **switch** can connect several devices, and **discovers** the MAC addresses.
- When a MAC address has been discovered, a switch can distinctively **forward messages** to the associated device.

10 / 53

Assigning an Internet address



11 / 53

Structure of an IP address

32 bits

network identifier	host identifier

- An IP address consists of a **network identifier** and a **host identifier**

  Network ID: worldwide unique address of a (small area) network to which messages can be **routed**

  Host ID: network-wide unique address associated with a device/host

- In the Internet, messages are always routed to a network. Internal routers handle subsequent forwarding to the hosts/devices using host IDs

12 / 53

IP addresses and home networks

Observation

Each home (or small organization) is assigned exactly one IP address.

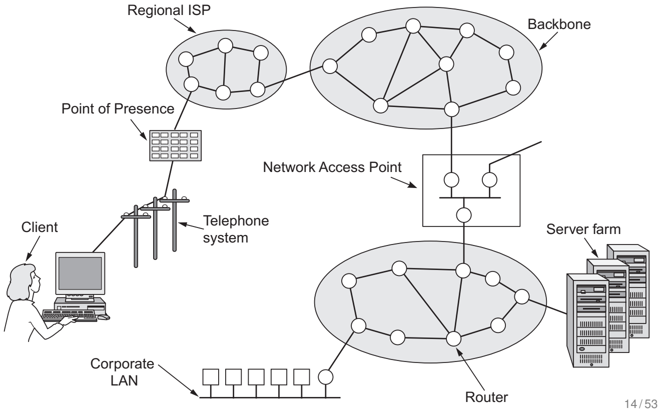
Note

Using a bag of tricks, we can **share** that address among different devices. For now, it is important to know that all your devices at home have (essentially) the same **external** IP address.

Consequence

All devices in a home network are seen **by the outside world** as being one and the same.

Large-area networks



Autonomous system

Description

An autonomous system is an organizational unit that maintains a collection of (interconnected) communication networks. An AS announces its **accessible** networks as ⟨*AS number*, *network identifier*⟩ pairs.

A simple number...

In 2009, there were approximately 25,000 ASes.

Measuring the AS topology

- Each AS i has a number of **border gateways**: a special router that can transfer messages between AS i and an AS to which that router is **linked**.
- If BG_1^i of AS i is linked to BG_1^j of AS $j \Rightarrow$ there is a physical connection between the two routers.
- Two gateways BG_1^i and BG_2^i of the same AS i, are always **internally linked**: they know how to reach each other through a communication path.
- A border gateway BG_1^i of AS i, attached to network n_i, announces $\langle i, n_i \rangle$ to its neighboring gateways.
- Assume BG_1^j of AS j is linked to $BG_1^i \Rightarrow BG_1^j$ can then announce that it knows a path to n_i: $\langle j, i, n_i \rangle$.
- Each other gateway BG_2^j of AS j can announce $\langle j, i, n_i \rangle$ to *its* linked neighbors.

16 / 53

Measuring the AS topology

Important observations

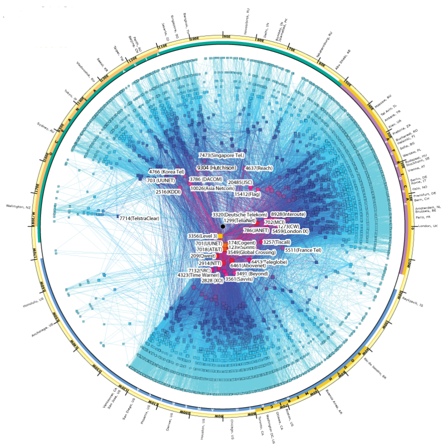
- Gateways store and announce **entire paths** to destinations.
- For proper routing, each gateway needs to store paths to **every network in the Internet**.

Conclusion

If we read the routing tables from only a few gateways, we should be able to obtain a reasonable complete picture of the **AS topology of the Internet**.

17 / 53

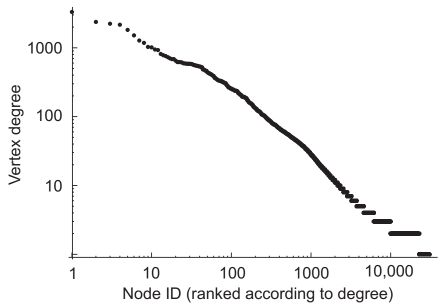
Visualizing the AS topology (CAIDA)



18 / 53

Example topology: October 2008

- Over 30,000 registered autonomous systems (including "double" registeries).
- Over 100,000 edges. **Note:** we may be missing more than 30% of all existing links!



19 / 53

Example topology: October 2008

Rank:	1	2	3	4	5	6	7	8	9	10
Degree:	3309	2371	2232	2162	1816	1512	1273	1180	1029	1012

Some observations

- Very high clustering coefficient for top-1000 hubs: an almost complete graph!
- Most paths no longer than 3 or 4 hops.
- Most ASes separated by shortest path of max. length 6.

20 / 53

Peer-to-peer overlay networks

Issue

Large-scale **distributed computer systems** are spread across the Internet, yet their constituents need to communicate directly with each other $\Rightarrow$ organize the system in an **overlay network**.

Overlay network

Collection of **peers**, where each peer maintains a **partial view** of the system. View is nothing but a list of other peers with whom communication connections can be set up.

Observation

Partial views may change over time $\Rightarrow$ an ever-changing overlay network.

21 / 53

Structured overlay network: Chord

Basics

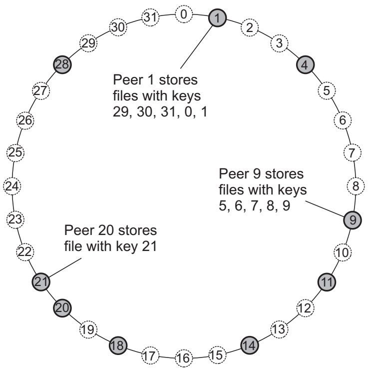
- Each peer is assigned a unique **m-bit identifier** id.
- Every peer is assumed to store data contained in a file.
- Each file has a unique **m-bit key** k.
- Peer with smallest identifier $id \geq k$ is responsible for storing file with key k.
- $succ(k)$: The peer (i.e., node) with the smallest identifier $p \geq k$.

Note

All arithmetic is done modulo $M = 2^m$. In other words, if $x = k \cdot M + y$, then $x \bmod M = y$.

22 / 53

Example



23 / 53

Efficient lookups

Partial view = finger table

- Each node p maintains a **finger table** $FT_p[]$ with at most m entries:

$$FT_p[i] = succ(p + 2^{i-1})$$

Note: $FT_p[i]$ points to the first node succeeding p by at least 2^{i-1}.

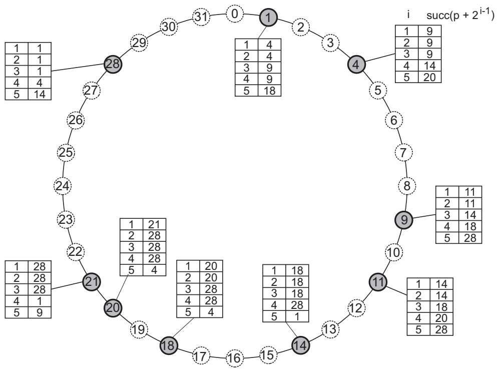
- To look up a key k, node p forwards the request to node with index j satisfying

$$q = FT_p[j] \leq k < FT_p[j+1]$$

- If $p < k < FT_p[1]$, the request is also forwarded to $FT_p[1]$

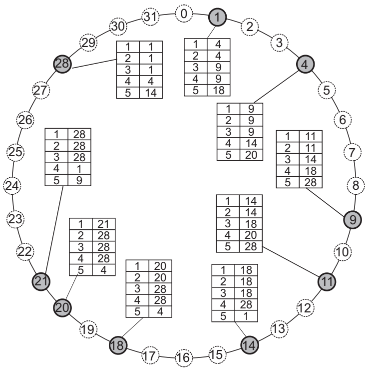
24 / 53

Example finger tables



25 / 53

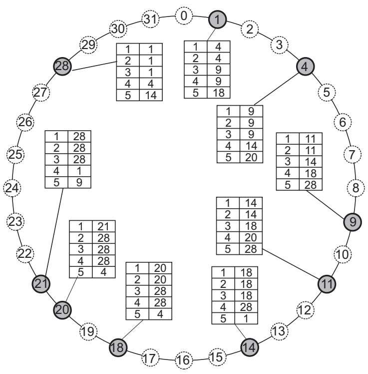
Example lookup: 15@4



1. $FT_4[4] \leq 15 < FT_4[5]$
   $\Rightarrow 4 \rightarrow 14$
2. $p = 14 < 15 < FT_p[1]$
   $\Rightarrow 14 \rightarrow 18$

26 / 53

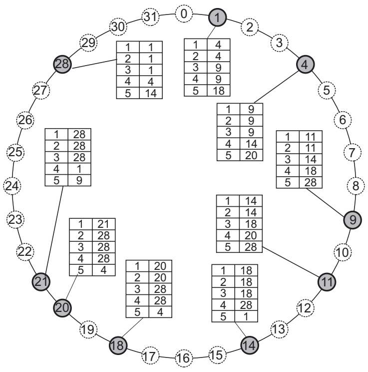
Example lookup: 22@4



1. $FT_4[5] \leq 22$
   $\Rightarrow 4 \rightarrow 20$
2. $FT_{20}[1] \leq 22 < FT_{20}[2]$
   $\Rightarrow 20 \rightarrow 21$
3. $p = 21 < 22 < FT_{21}[1]$
   $\Rightarrow 21 \rightarrow 28$

27 / 53

Example lookup: 18@20



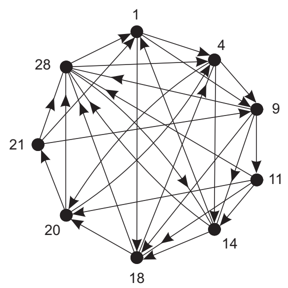
1. $p = 20 \not< 18 < FT_p[1]$
   $\not\Rightarrow 20 \rightarrow 21$
2. $FT_{20}[5] < 18$
   $\Rightarrow 20 \rightarrow 4$
3. $FT_4[4] \leq 18 < FT_4[5]$
   $\Rightarrow 4 \rightarrow 14$
4. $p = 14 < 18 < FT_p[1]$
   $\Rightarrow 14 \rightarrow 18$

28 / 53

The Chord graph

Essence

Each peer represented by a vertex; if $FT_p[i] = j$, add arc $\langle \overrightarrow{i,j} \rangle$, but keep directed graph strict.

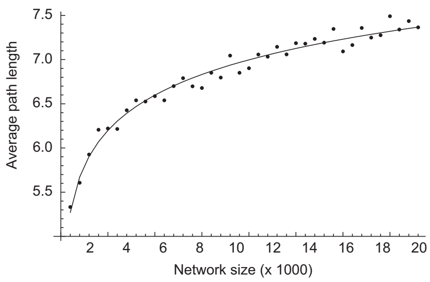


29 / 53

Chord: path lengths

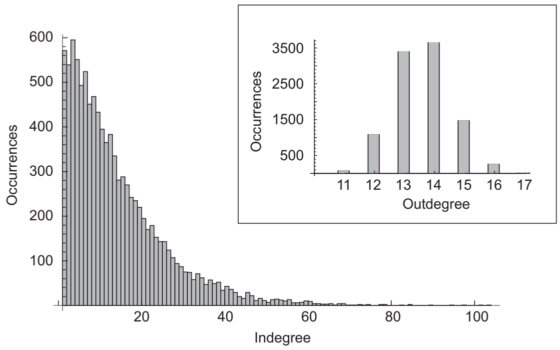
Observation

With $d_2^n(i,j) = \min\{|i-j|, n-|i-j|\}$, we can see that every peer is joined with another peer at distance $\frac{1}{2}n, \frac{1}{4}n, \frac{1}{8}n, \ldots, 1$.



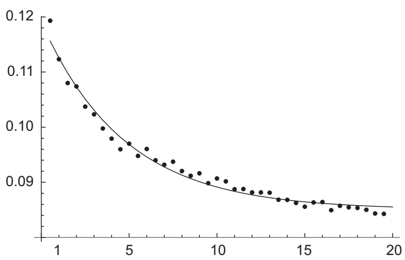
30 / 53

Chord: degree distribution



31 / 53

Chord: clustering coefficient



Note

CC is computed over undirected Chord graph; x-axis shows number of 1000 nodes.

32 / 53

Epidemic-based networks

Basics

Consider a collection of peers $\mathbf{P} = \{p_1, \ldots, p_n\}$. Each peer can store lots of files. Each file f has a **version** $v(f)$. The **owner** of f is a single, unique peer, $own(f)$ who can update f.

Goal

We want to propagate updates of file f through a network of peers. $v(f,p)$ denotes version of file f at peer p. $FS(p)$ is set of files at peer p. If $f \notin FS(p) \Rightarrow v(f,p) = 0$.

$$\forall f,p : v(f, own(f)) \geq v(f,p)$$

33 / 53

Epidemic protocol

The core

Each peer $p \in \mathbf{P}$ **periodically** does the following:

1. $\forall f \in FS(p) : v(f,p) > v(f,q) \Rightarrow FS(q) \leftarrow FS(q) \cup \{f@p\}$
2. $\forall f \in FS(q) : v(f,p) < v(f,q) \Rightarrow FS(p) \leftarrow FS(p) \cup \{f@q\}$

34 / 53

General framework

Active part

```
repeat
    wait T
    q ← select 1 from PV_p
    R_p ← select s from PV_p
    send R_p ∪ {p} \ {q} to q
    skip
    receive R^p_q from q
    PV_p ← select m from PV_p ∪ R^p_q
until forever
```

Passive part

```
repeat
    skip
    skip
    skip
    receive R^q_p from any p
    R_q ← select s from PV_q
    send R_q ∪ {q} \ {p} to p
    PV_q ← select m from PV_q ∪ R^q_p
until forever
```

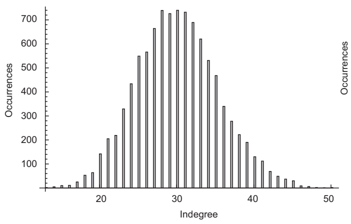

35 / 53

Newscast

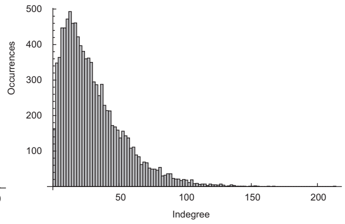
Issue	Policy	Description
view size	$m = 30$	Each partial view has size 30
peer selection	random	Each peer uniformly at random selects a peer from its partial view
reference selection	random	A random selection of s peers is selected from a partial view to be exchanged with the selected peer
view size reduction	random	If the view size has grown beyond m, a random selection of references is removed to bring it back to size m

36 / 53

Newscast: evolution indegree distribution



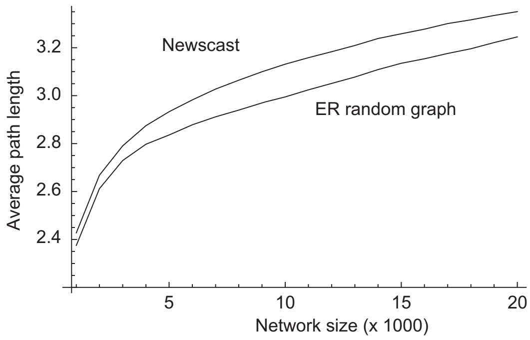
Initially



After 200 rounds

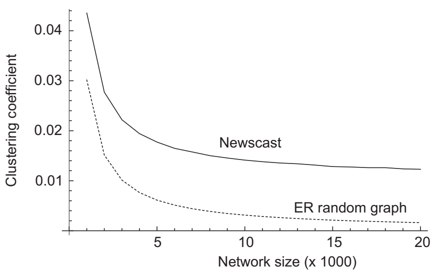
37 / 53

Newscast: evolution path length



38 / 53

Newscast: evolution cluster coefficient



Question

For which kind of $ER(n, p)$ graphs is this a fair comparison?

39 / 53

The Web

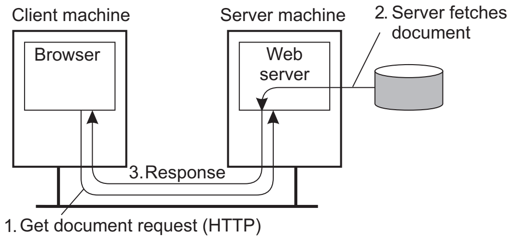
Web basics

- **Simple view**: the Web consists of **hyperlinked documents**.
- Hyperlinked: document *A* carries a reference to document *B*. When reference is **activated**, browser fetches document *B*.
- Collection of documents forms a **site**, with its own associated **domain name**.

Some numbers

It has been estimated that by 2008, there were at least 75 million Web sites from which Google had discovered more than a trillion Web pages.

Web basics



Measuring the topology of the Web

Problem

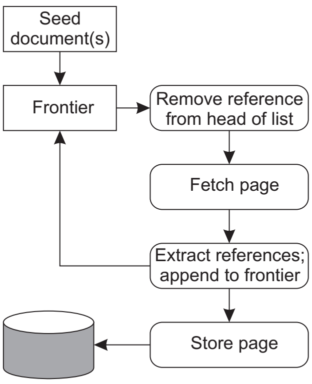
With an estimated size of over a trillion Web pages, pages coming and going, and links changing all the time, how can we ever get a **snapshot** of the Web? **We can't**.

Practical issue: crawling the Web

In order to measure anything, we need to be able to identify pages and the links that refer to them.

43 / 53

Web crawler



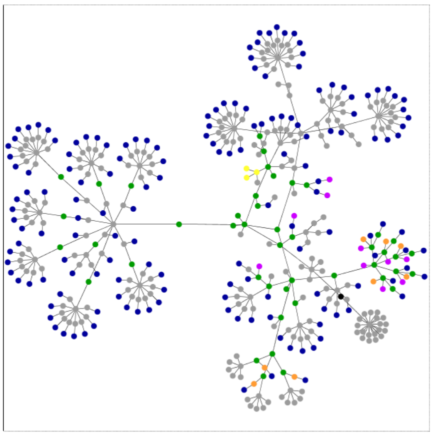
- Start with seed pages
- Store pages to inspect in frontier
- Analyze page and store found references in frontier

Observation

Using seed documents, we are accessing the **reachable pages**.

44 / 53

Webpages as graphs: The VU website



45 / 53

Sampling the Web

Observation

The Web is so huge, that we can only hope to **draw a reasonable sample**, and hope that this sample represents the structure of the actual Web. **We are asking for trouble**.

Starting point

Let us try to represent the Web as a **bowtie**:

SCC: $\forall v, w \in SCC, \exists(v, w)$-path of hyperlinks.

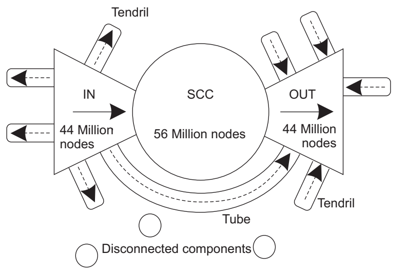
IN: $\forall v \in IN, w \in SCC : \exists(v, w)$-path, but no (w, v)-path.

OUT: $\forall v \in SCC, w \in OUT : \exists(v, w)$-path, but no (w, v)-path.

TENDRILS: Essentially: the rest.

46 / 53

The Web as a bowtie: Starting from AltaVista



47 / 53

Sampling the Web

Observation

It turns out that for different seeds, we do obtain different bowties.

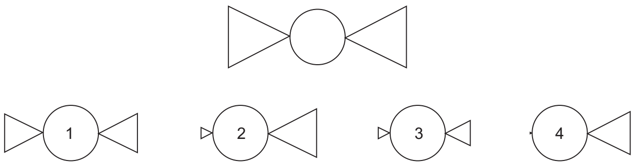
Component	Sample 1	Sample 2	Sample 3	Sample 4
SCC	56.46%	65.28%	85.87%	72.30%
IN	17.24%	1.69%	2.28%	0.03%
OUT	17.94%	31.88%	11.26%	27.64%
Other	8.36%	1.15%	0.59%	0.02%
Total size	80.57M	18.52M	49.30M	41.29M

48 / 53

Sampling the Web

Component	Sample 1	Sample 2	Sample 3	Sample 4
SCC	56.46%	65.28%	85.87%	72.30%
IN	17.24%	1.69%	2.28%	0.03%
OUT	17.94%	31.88%	11.26%	27.64%
Other	8.36%	1.15%	0.59%	0.02%
Total size	80.57M	18.52M	49.30M	41.29M

AltaVista

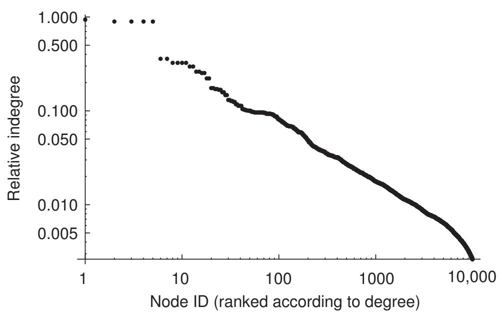


Question

Which conclusion can we draw from these samples?

49 / 53

Web graphs: indegree distribution



Observation

It turns out that $\mathbb{P}[\delta_{in} = k] \propto \frac{1}{k^{2.1}} \Rightarrow$ another scale-free network.

50 / 53

Side step: Google's PageRank

Observation

Google uses hyperlinks *to* a page p as a criterion for the *importance* of a page:

$$rank(p) = (1-d) + d \sum_{\langle \overrightarrow{q,p} \rangle \in E} \frac{rank(q)}{\delta_{out}(q)}$$

where $d \in [0,1)$ is a constant (probably 0.85 in the case of Google).

Question

This is a **recursive** definition. What's going on?

51 / 53

Side step: Google's PageRank

Observation

PageRank is clearly based on indegrees, yet the rank of a page and its indegree turn out to be only weakly correlated.

Observation

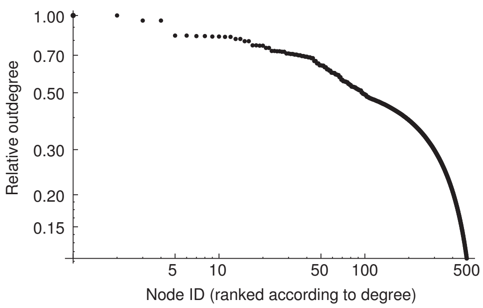
When we rank pages according to PageRank: $\mathbb{P}[rank = k] \propto \frac{1}{k^{2.1}}$

Observation

Characterizing and sampling the Web is again seen to be far from trivial.

52 / 53

Web graphs: oudegree distribution



Observation

To analyze the Web graph, we need to be very careful regarding measurements and conclusions.

53 / 53