

Distributed Systems

(4th edition, version 01)

Chapter 09: Security

Dependability

Basics

A dependable system provides availability, reliability, safety, maintainability, confidentiality, and integrity.

- **Confidentiality**: refers to the property that information is disclosed only to authorized parties.
- **Integrity**: alterations to a system's assets can be made only in an authorized way, ensuring accuracy and completeness.

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Alternative

We attempt to protect against **security threats**:

1. Unauthorized information disclosure (**confidentiality**)
2. Unauthorized information modification (**integrity**)
3. Unauthorized denial of use (**availability**)

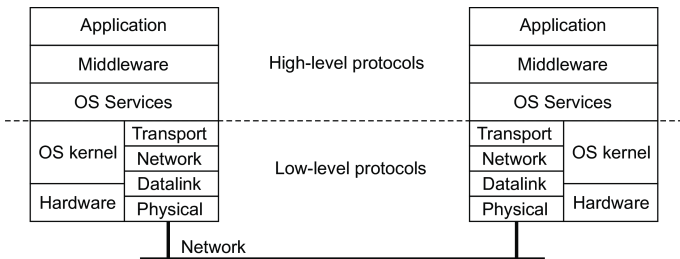
Security mechanisms

- **Encryption**: transform data to something an attacker cannot understand, or that can be checked for modifications.
- **Authentication**: verify a claimed identity.
- **Authorization**: check an authenticated entity whether it has the proper rights to access resources.
- **Monitoring and auditing**: (continuously) trace access to resources

Security principles

- **Fail-safe defaults**: defaults should already provide good protection.
Infamous example: the default “*admin,admin*” for edge devices.
- **Open design**: **do not** apply security by obscurity: every aspect of a distributed system is open for review.
- **Separation of privilege**: ensure that critical aspects of a system can never be fully controlled by just a single entity.
- **Least privilege**: a process should operate with the fewest possible privileges.
- **Least common mechanism**: if multiple components require the same mechanism, then they should all be offered the same implementation of that mechanism.

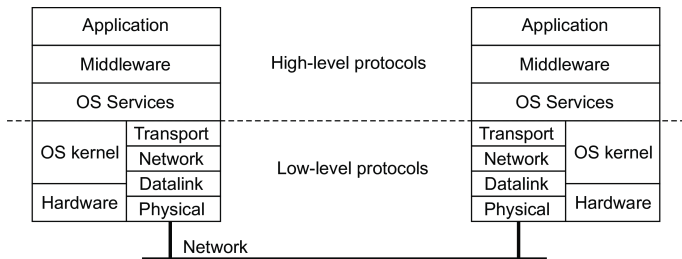
Where to implement security mechanisms?



Observation

We are increasingly seeing **end-to-end** security, meaning that mechanisms are implemented at the level of applications.

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We are increasingly seeing **end-to-end** security, meaning that mechanisms are implemented at the level of applications.

Issue: which layer do we trust?

Trusted Computing Base: The set of all security mechanisms in a (distributed) computer system that are necessary and sufficient to enforce a security policy.

On privacy

Observation

Privacy and confidentiality are closely related, yet are different. **Privacy** can be **invaded**, whereas **confidentiality** can be **breached** \Rightarrow ensuring confidentiality is not enough to guarantee privacy.

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Right to privacy

The right to privacy is about “a right to **appropriate** flow of personal information.” Control who gets to see what, when, and how \Rightarrow a person should be able to stop and revoke a flow of personal information.

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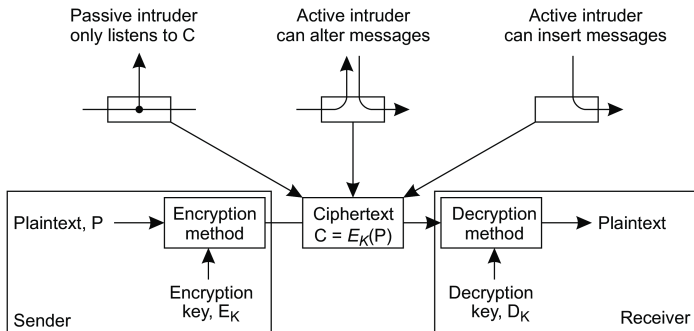
General Data Protection Regulation (GDPR)

The GDPR is a comprehensive set of regulations aiming to **protect personal data**.

GDPR: Database perspective

GDPR regulation	Impact on database systems	
	Attributes	Actions
Collect data for explicit purposes	Purpose	Metadata indexing
Do not store data indefinitely	TTL	Timely deletion
Inform customers about GDPR metadata associated with their data	Purpose, TTL, Origin, Sharing	Metadata indexing
Allow customers to access their data	Person id	Metadata indexing
Allow customers to erase their data	TTL	Timely deletion
Do not use data for objected reasons	Objections	Metadata indexing
Allow customers to withdraw from algorithmic decision-making	Automated decisions	Metadata indexing
Safeguard and restrict access to data		Access control
Do not grant unlimited access to data		Access control
Audit operations on personal data	Audit trail	Monitor and log
Implement appropriate data security		Encryption
Share audit trails from affected systems	Audit trail	Monitor and log

Cryptography



Basic concepts

- **Plaintext**: the original message or data (P)
- **Ciphertext**: the encrypted version of the the plaintext (C)
- **Encryption key**: input E_K to a function for encryption: $C = E_K(P)$
- **Decryption key**: input D_K to a function for decryption: $P = D_K(C)$

Cryptosystems

Symmetric : if $P = D_K(E_K(P))$ then $D_K = E_K$.

Asymmetric : if $P = D_K(E_K(P))$ then $D_K \neq E_K$.

Also called **public-key systems** with a **publicly known** key PK and **secret key** SK

Examples

Let PK_X denote public key of X and SK_X the associated secret key.

Confidential message : if m is to be kept private: $C = PK_{receiver}(m)$.

Authenticated message : if m is to be authenticated: $C = SK_{sender}(m)$.

Homomorphic encryption

Mathematical operations on plaintext can be performed on the corresponding ciphertext: if x and y are two numbers, then

$$E_K(x) \star E_K(y) = E_K(x \star y)$$

Hash functions

Description

A hash function H takes a message m of arbitrary length as input and produces a bit string h having a fixed length as output:

$$h = H(m) \text{ with length of } h \text{ fixed.}$$

Example: digital signature

Alice computes a digest from m ; encrypts the digest with her private key; encrypted digest is sent along with m to Bob:

$$\text{Alice: send } [m, \text{sig}] \text{ with } \text{sig} = SK_A(H(m)).$$

Bob decrypts digest with Alice's public key; separately calculates the message digest. If both match, Bob knows the message has been signed by Alice:

$$\text{Bob: receive } [m, \text{sig}], \text{ compute } h' = H(m) \text{ and verify } h' = PK_A(\text{sig}).$$

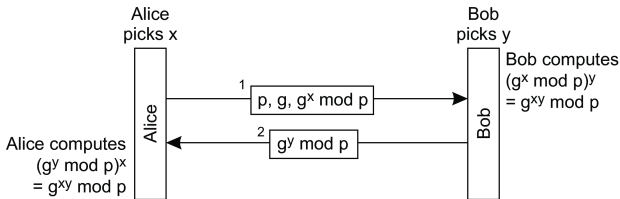
Key management

Essence

How do Alice and Bob get the correct (often shared) keys so that they can set up secure channels?

Diffie-Hellman key exchange

Assume two large, nonsecret numbers p and g (with specific mathematical properties):



DH key exchange: example

Multiparty computation

Can we **protect private data** while **computing statistics**? Who has the highest salary without revealing salaries? Can we compute the number of votes cast for a specific candidate without revealing who voted for whom?

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Oblivious transfer

Alice has n **secret messages** m_1, \dots, m_n . Bob is interested (and allowed) to **know only message** m_i . Which message he wants to know should be kept secret to Alice; all messages $m_j \neq m_i$ should be kept secret to Bob.

DH key exchange: example

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Oblivious transfer

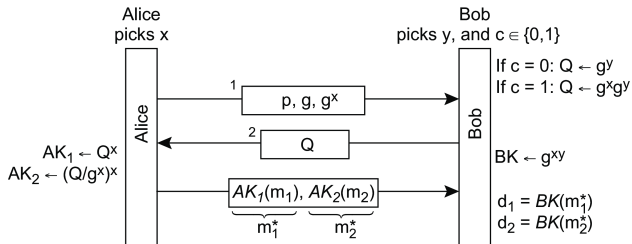
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Solution

Bob generates a number Q that Alice, in turn, uses to generate n different **encryption keys** PK_1, \dots, PK_n : $m_i^* = PK_i(m_i)$

Bob uses Q to generate a **decryption key** SK_i that matches **only** PK_i . When Bob receives m_1^*, \dots, m_n^* he can decrypt only m_i^* . $SK_i(m_j^*)$ (with $i \neq j$) will fail.

1-out-of-2 oblivious transfer



Analysis

- $c = 0 \Rightarrow Q = g^y, AK_1 = BK = g^{xy}, AK_2 = g^{xy-x^2}.$
- $c = 1 \Rightarrow Q = g^{x+y}, AK_1 = g^{x^2+xy}, AK_2 = BK = g^{xy}.$

Example, continued

Preliminaries

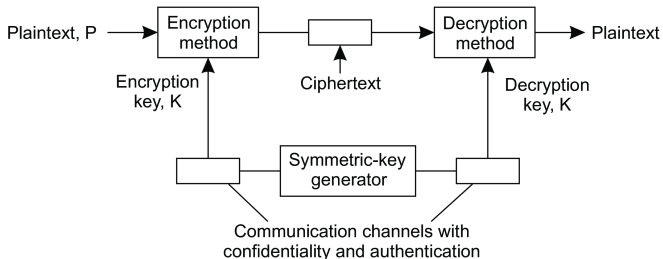
- P_1 and P_2 need to compute $F(a, b)$.
- Parameter a is secret and known only to P_1 ; secret b known only to P_2 .
- $a \in \mathbf{X}$ and $b \in \mathbf{Y}$; \mathbf{X} and \mathbf{Y} are finite.
- Construct a $|\mathbf{X}| \times |\mathbf{Y}|$ matrix \mathbf{F} .
- $\mathbf{F}[i, j] = F(x_i, y_j)$ for each pair $(x_i, y_j) \in \mathbf{X} \times \mathbf{Y}$.

Solution

- P_1 generates $|\mathbf{X}| \cdot |\mathbf{Y}|$ unique key pairs (K_i, K_j)
- Construct $\mathbf{F}^*[i, j] = K_i(K_j(F(x_i, x_j)))$. Assume $a = x_i$.
- P_1 permutes \mathbf{F}^* and sends it along with K_i to P_2
- P_1 sends Q using a 1-out-of- $|\mathbf{Y}|$ oblivious transfer.
- Assume $b = y_j$. Using Q , P_2 can construct K_j , and only K_j
- P_2 decrypts $\mathbf{F}^*[i, j]$, corresponding to $F(a, b)$.

What is needed to distribute keys

Symmetric-key distribution

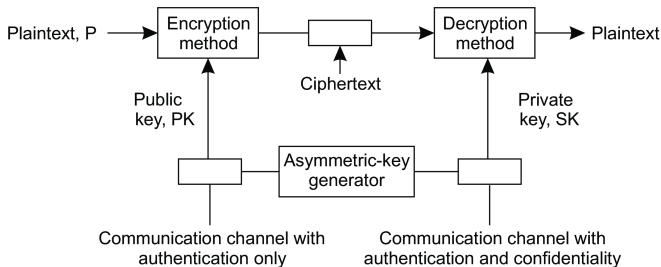


Observation

In general, we will need a **secure channel** to distribute the secret key to the communicating parties.

What is needed to distribute keys

Public-key distribution



Observation

No need for a secure channel in the case of the public key, but you do need to know that the key is **authentic** \Rightarrow have the public key be **signed** by a **certification authority**. Note, we do need to trust that authority, or otherwise make sure that its signature can be verified as well.

Authentication

Essence

Verifying the claimed identity of a person, a software component, a device, and so on.

Means of authentication

1. Based on what a client **knows**, such as a **password** or a **personal identification number**.
2. Based on what a client **has**, such as an **ID card**, **cell phone**, or **software token**.
3. Based on what a client **is**, i.e., static biometrics such as a **fingerprint** or **facial characteristics**.
4. Based on what a client **does**, i.e., dynamic biometrics such as **voice patterns** or **typing patterns**.

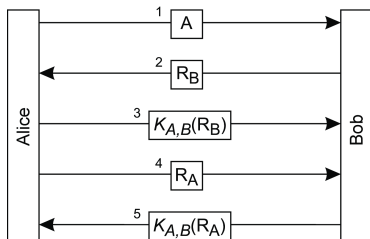
Authentication versus message integrity

Observation

Authentication without integrity (and *vice versa*) is meaningless:

- Consider a system that supports authentication but no mechanisms to ensure message integrity. Bob may know for sure that Alice sent m , but how useful is that if he doesn't know that m may have been modified?
- Consider a system that guarantees message integrity, but does not provide authentication. Can Bob be happy with a guaranteed unmodified message that states he just won \$1,000,000?

Using a shared secret key

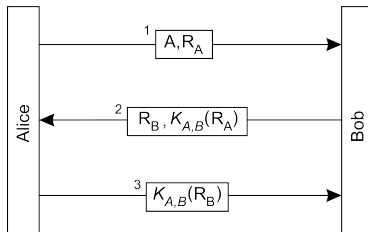


Steps

1. Alice announces she wants to talk to Bob.
2. Bob returns a **nonce**.
3. Alice encrypts the nonce with the shared key $K_{A,B}$, thus proving that she owns $K_{A,B} \Rightarrow$ **Bob knows he's talking to Alice**.
4. Alice sends a nonce to Bob.
5. Bob returns proof that he owns the shared secret key as well \Rightarrow **Alice knows she's talking to Bob**.

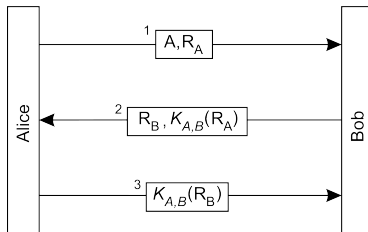
About optimizations

Let's reduce the number of messages

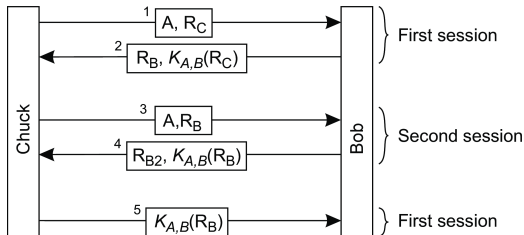


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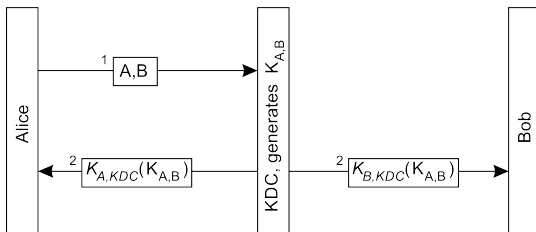
Let's reduce the number of messages



We just broke the protocol



Using a Key Distribution Center

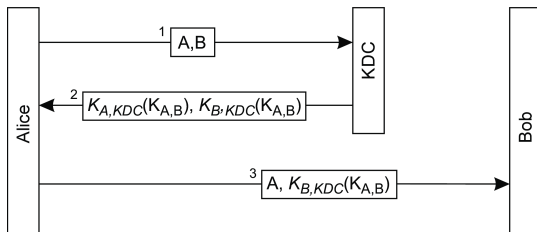


Basics

Every client has a secret key shared with the KDC.

1. Alice tells the KDC that she wants to talk to Bob
2. The KDC sends a fresh secret key, shared by Alice and Bob

Using a Key Distribution Center

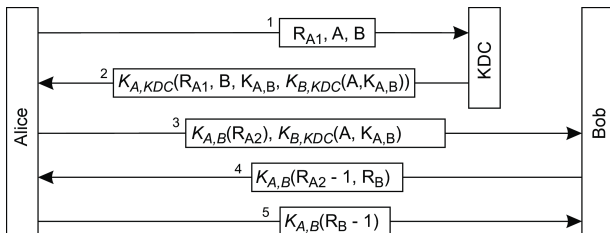


Basics

Using a **ticket** is practically better:

1. Alice tells the KDC that she wants to talk to Bob
2. The KDC sends a fresh secret key, shared by Alice and Bob
3. Alice tells Bob that she wants to talk, along with the key to be used.

The Needham-Schroeder protocol



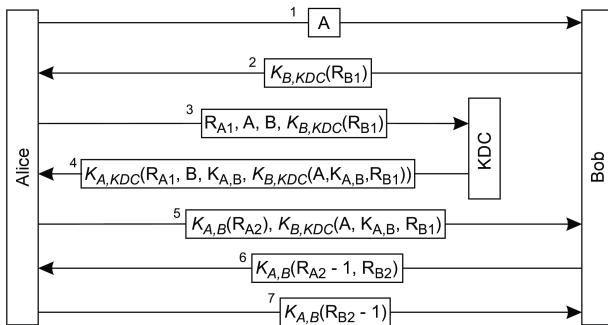
Important observation

In the case of request-response messages, you want to make sure that the received response, is associated with the sent request. Mitigates **replay attacks**.

General principle

Use **nonces** to relate any combination of request-response messages.

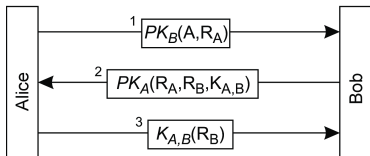
Mitigate against reuse of keys



Some observations

- Note how $B1$ ties message #2 to #5
- Note that by returning $R_{A2} - 1$ in #6, Bob proves he knows $K_{A,B}$
- And, likewise, in the case of Alice in #6 (by modifying R_{B2}).

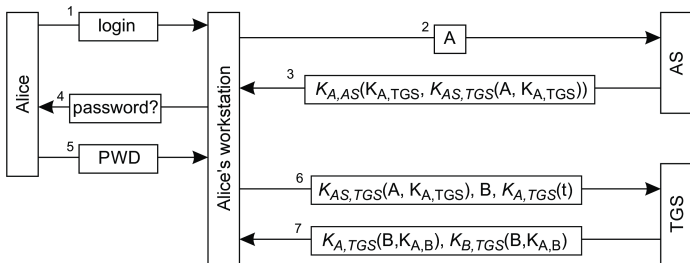
Using public keys



Steps

1. Alice tells Bob she wants to talk, sending a nonce R_A , and encrypting the message with Bob's public key.
2. Bob generates a **shared secret session key** $K_{A,B}$, **proves he is the owner** of PK_B by decrypting R_A , and challenges Alice to prove she owns PK_A .
3. Alice decrypts the response, and **proves to Bob that she is Alice** by then sending Bob's nonce back encrypted with the generated session key $K_{A,B}$.

Practical example: Kerberos



Essence

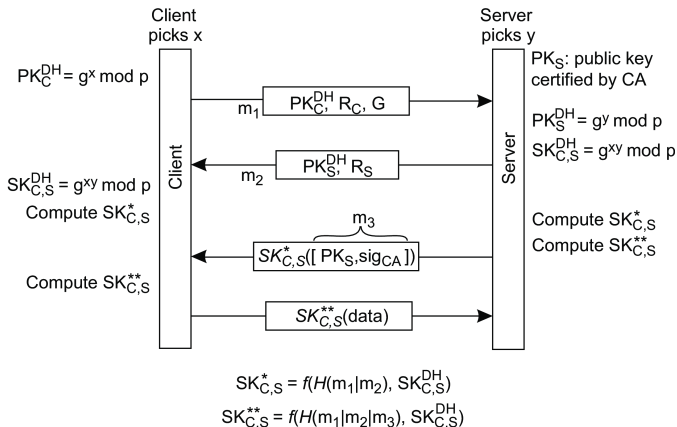
1,2 Alice types in her login name.

3 The **Authentication Service** returns a **ticket** $K_{AS,TGS}(A, K_{A,TGS})$ that she can use with the **Ticket Granting Service**.

4,5 To be able to decrypt the message, Alice must type in her password. She is then logged in. Using the AS in this way, we have a **single sign-on** system.

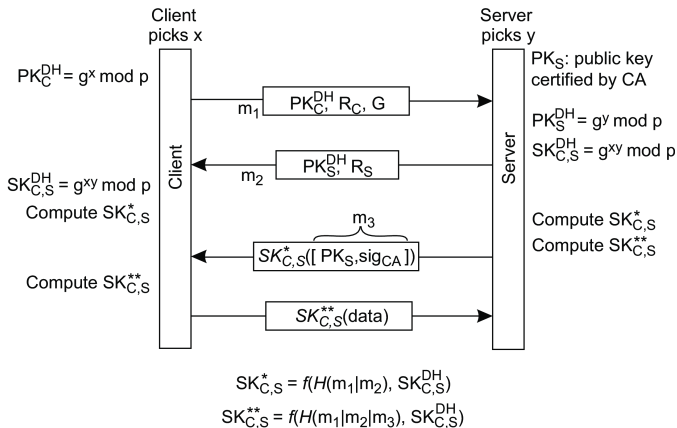
6,7 Alice wants to talk to Bob, and requests the TGS for a session key.

Transport Layer Security



- G denotes a specific set of parameter settings, called a **group** (e.g., values for p and g).

Transport Layer Security



- The client uses a **nonce** R_C ; the server uses R_S
- $H(m_1|m_2)$ denotes the hash over the concatenation of m_1 and m_2

On trust

Definition

Trust is the assurance that one entity holds that another will perform particular actions according to a specific expectation.

Important observation

- Expectations have been made explicit \Rightarrow no need to talk about trust?
- **Example:** Consider a Byzantine fault-tolerant process group of size n
 - **Specification:** the group can tolerate that at most $k \leq (n-1)/3$ processes go rogue.
 - **Realisation:** for example PBFT.
 - **Consequence:** if more than k processes fail, all bets are simply off.
 - **Consequence:** it's not about trust, it's all about meeting specifications.
- **Observation:** if a process group often does not meet its specifications, one may start to doubt its **reliability**, but this is something else than (dis)trusting the system.

Sybil attack

Essence: Just create multiple identities, but owned by one entity

- In the case of a peer-to-peer network:

```
1 H = set of honest nodes
2 S = set of Sybil nodes
3 A = Attacker node
4 d = minimal fraction of Sybil nodes needed for an attack
5
6 while True:
7     s = A.createNode()      # create a Sybil node
8     S.add(s)                # add it to the set S
9
10    h = random.choice(H)     # pick an arbitrary honest node
11    s.connectTo(h)           # connect the new sybil node to h
12
13    if len(S) / len(H) > d:  # enough sybil nodes for...
14        A.attack()           # ...an attack
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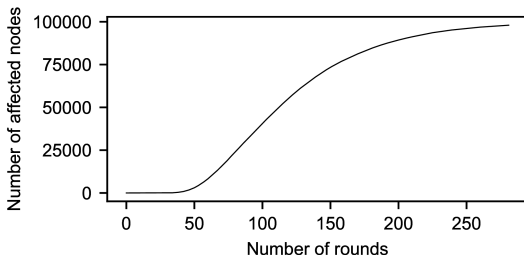
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```

- In the case of a Web-of-trust:
 - Endorse a public key without an out-of-band check.
 - *Bob* checks with $k > 1$ others that they have endorsed *Alice's* key.
 - *Alice* creates $k > 1$ identities each stating her key is valid.

Eclipse attack

Essence: Try to isolate a node from the network

Example: a **hub attack** in the case of a gossip-based service. In this case, when exchanging links to other peers, a colluding node returns links only to other colluders.



Affected node: has links only to colluders.

General solution

Use a centralized certification authority.

Preventing Sybil attacks: Blockchain solutions

Essence: creating an identity comes at a cost

In the case of **permissionless blockchains**:

- **Proof-of-Work**: Let validators run a computational race. This approach requires considerable **computational resources**
- **Proof-of-Stake**: Pick a validator as a function of the number of tokens it owns. This approach requires risking **loss of tokens**.

Preventing Sybil attacks: Decentralized accounting

A simple example

- Each node P maintains a list of nodes interested in doing work for P : the **choice set** of P ($choice(P)$).
- Selecting $Q \in choice(P)$ depends on Q 's work for others (i.e., its **reputation**).
- P maintains a (**subjective**) view on reputations. Of course, P knows precisely what it has done for others, and what others have done for P .
- P can compute a **capacity** ($cap(Q)$):

$$cap(Q) = \max\{MF(Q, P) - MF(P, Q), 0\}$$

with $MF(P, Q)$ the amount of work that P has, or could have contributed to work done for Q , including the work done by others.

Preventing Sybil attacks: Decentralized accounting

Essence: Keep track of work that nodes do for each other

- Assume R directly contributed 3 units of work for Q , and R had processed 7 units for $P \Rightarrow P$ may have contributed 3 units of work for Q , through R .
- **Reasoning:** R may never have been able to work for Q , if it had not worked for P .

Preventing Sybil attacks: Decentralized accounting

How Sybil attacks are prevented

- Let $Q \in \text{choice}(P)$ create n Sybil nodes Q_1^*, \dots, Q_n^* ; $Q = Q_0^*$
- For work by Q_i^* for Q_j^* to increase $\text{cap}(Q_j^*)$:
 1. Q_j^* needs to have worked for some node R
 2. R needs to have worked for P

In other words: Q can successfully attack only if it had worked for honest nodes. Also, honest nodes have to work for Q : the total capacity $Tcap(Q)$ of the Sybils must grow, with

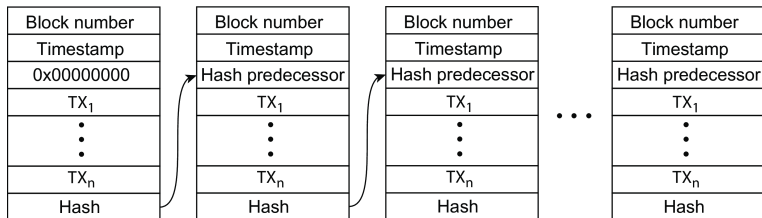
$$Tcap(Q) = \sum_{k=0}^n \text{cap}(Q_k^*)$$

- Assume that P works 1 unit for $Q_i^* \Rightarrow MF(P, Q_i^*)$ increases by 1 unit $\Rightarrow \text{cap}(Q_i^*)$ drops by 1 unit, and so does $Tcap(Q)$.
- As soon as $Tcap(Q)$ drops to 0, P will look at other nodes.

Trusting a system: Blockchains

Essence

One needs to know for sure that the information in a blockchain has not been tampered with: **data integrity assurance**. **Solution**: make sure that no change can go unnoticed (recall: a blockchain is an append-only data structure).



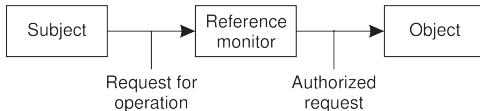
Observation

Any change of block B_k , will affect its hash value, and thus that of B_{k+1} , which would then also need to be changed, in turn affecting the hash value of B_{k+2} , and so on.

Access control: General model

Authorization

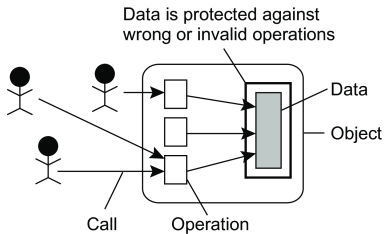
Making sure that authenticated entities have only access to specific resources.



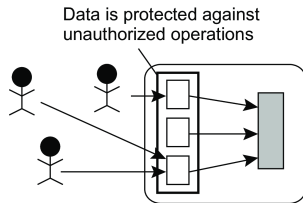
Observation

The reference monitor needs to be **tamperproof**: it is generally implemented under full control of the operating system, or a secure server.

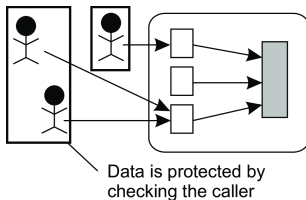
Protection



...against invalid operations



...against unauthorized access



...against unauthorized invokers

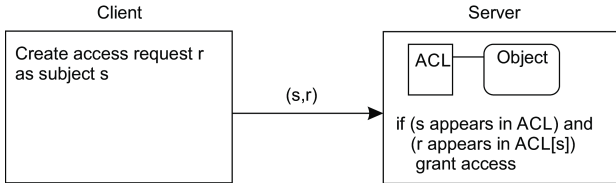
Access control policies

1. **Mandatory access control**: A central administration defines who gets access to what.
2. **Discretionary access control**: The owner of an object can change access rights, but also who may have access to that object.
3. **Role-based access control**: Users are not authorized based on their identity, but based on the role they have within an organization.
4. **Attribute-based access control**: Attributes of users and of objects they want to access are considered for deciding on a specific access rule.

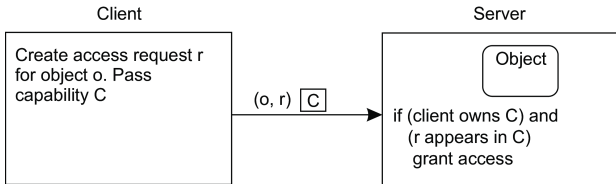
Access control matrix

Theory

Construct a matrix in which $M[s, o]$ describes the access rights subject s has with respect to object o . **Impractical**, so use **access control lists** or **capabilities**.



Access control list



Capabilities

Special case: Attribute-based Access Control

Distinguish different classes of attributes:

- **User attributes:** name, data of birth, current roles, home address, department, qualifiers obtained, contract status, etc. May also depend on role (e.g., teacher or student).
- **Object attributes:** anything – creator, last-modified time, version number, file type, file size, but also information related to its content.
- **Environmental attributes:** describe the current state of the system, e.g., date and time, current workload, maintenance status, storage properties, available services, etc.
- **Connection attributes** provide information on the current session, e.g., IP address, session duration, available bandwidth and latency estimates, type and strength of security used.
- **Administrative attributes:** reflect global policies, e.g., minimal security settings, general access regulations, and maximum session durations.

Example: the Policy Machine

Essence

A server maintains sets of (*attribute,value*) pairs, distinguishing **users**, **applications**, **operations**, and **objects**. At the core, we formulate **access control rules**.

Access control rules

- **Assignment**: A user u can be assigned to an attribute ua : $u \rightarrow ua$. An object to an attribute: $o \rightarrow oa$; an attribute to an attribute: $ua_1 \rightarrow ua_2$ (meaning that if $u \rightarrow ua_1$, then $u \rightarrow ua_2$. Leads to rules like $allowed(ua, ops, oa)$: users assigned to ua are allowed to execute operations in ops on objects assigned to oa .
- **Prohibition**: explicitly state what is **not** allowed, such as $denied(u, ops, os)$. Also: $denied(u, ops, \neg os)$, meaning denial when u wants to perform o assigned to ops on an object not in os .
- **Obligation**: automated action upon an event, such as denying copying of information:

when u reads $f \in fs$ then $denied(u, \{write\}, \neg fs)$.

Delegation

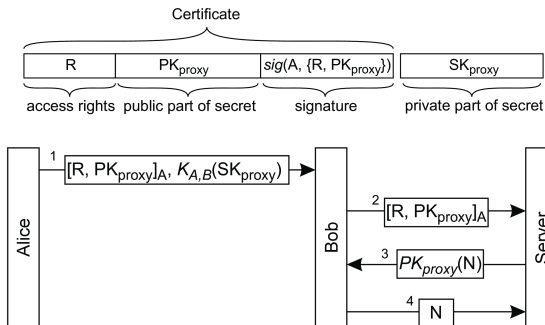
What's the issue?

Alice makes use of an e-mail service provider who stores her mailbox. She is required to log in to the provider to access her mail. Alice wants to use her own local mail client. How to allow that mail client to act on behalf of Alice? **How to delegate Alice's access rights to her mail client?**

Observation

It is not a good idea to hand over all user credentials to an application: why would the application or the machine be trusted? \Rightarrow use a **security proxy**.

Security proxy



How it works

1. Alice passes some rights R to Bob, together with a secret key SK_{proxy}
2. When Bob wants to exercise his rights, he passes the certificate
3. The server wants Bob to prove he knows the secret key
4. Bob proves he does, and thus that Alice had delegated R .

Example: Open Authorization (OAuth)

Four different roles

- **Resource owner**: typically an end user.
- **Client**: an application that one would like to act on behalf of the resource owner,
- **Resource server**: An interface through which a person would normally access the resource.
- **Authorization server**: an entity handing out certificates to a client on behalf of a resource owner.

Initial steps

1. The client application registers itself at the authorization server and receives its own identifier, *cid*.
2. Alice wants to delegate a list *R* of rights \Rightarrow

Client: *send* [*cid*, *R*, $H(S)$]

with a hash of a temporary secret *S*

Completing the process

Final steps

3. Alice is required to log in and confirm delegation R to the client.
4. Server sends a temporary authorization code AC to client.
5. Client requests a final **access token**:

Client: *sends* $[cid, AC, S]$.

Sending S to the authorization server allows the latter to verify the identity of the client (by computing $H(S)$).

The authorization server has now (1) verified that Alice wants to delegate access rights to the client, and (2) has verified the identity of the client \Rightarrow it returns an access token to the client.

Example: decentralized authorization

WAVE (and keeping it very simple)

Essence: Alice delegates rights to Bob, Bob delegates some of those rights to Chuck.

- When Check wants to exercise his rights, there should be no need for Alice or Bob to be online.
- No one but Alice, Bob, and Chuck need to be aware of the delegation.

Essentials

Alice delegates rights R to Bob, for which he creates a keypair (PK_B^R, SK_B^R) :

$$A \text{ sends: } PK_B^R(\underbrace{[R|SK_A^R]}_{m_1})$$

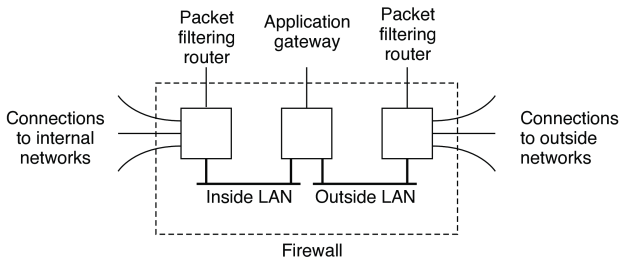
Bob delegates parts of those rights R' to Chuck, assuming he is allowed to do so:

$$B \text{ sends: } PK_C^{R'}(\underbrace{[R'|m_1|SK_B^R]}_{m_2})$$

Firewalls

Essence

Simply prevent anything nasty coming in, but also preventing unwanted outbound traffic.



Different types of firewalls

- **Packet-filtering gateway**: operates as a router and makes filters packets based on source and destination address.
- **Application-level gateway**: inspects the content of an incoming or outgoing message (e.g., gateways filtering spam e-mail).
- **Proxy gateway**: works as a front end to an application, filtering like an application-level gateway (e.g., Web proxies).

Intrusion detection systems

Two flavors

- **Signature-based**: matches against patterns of known network-level intrusions. Problematic when series of packets need to be matched, or when new attacks take place.
- **Anomaly-based**: assumes that we can model or extract typical behavior to subsequently detect nontypical, or anomalous behavior. Relies heavily on modern artificial-intelligence technologies.

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Using sensors

Key idea is to manage **false** and **true positives** (FP/TP) as well as **false** and **true negatives** (FN/TN). Maximize **accuracy** and **precision**:

$$\text{Accuracy: } \frac{TP + TN}{TP + TN + FP + FN}$$

$$\text{Precision: } \frac{TP}{TP + FP}$$